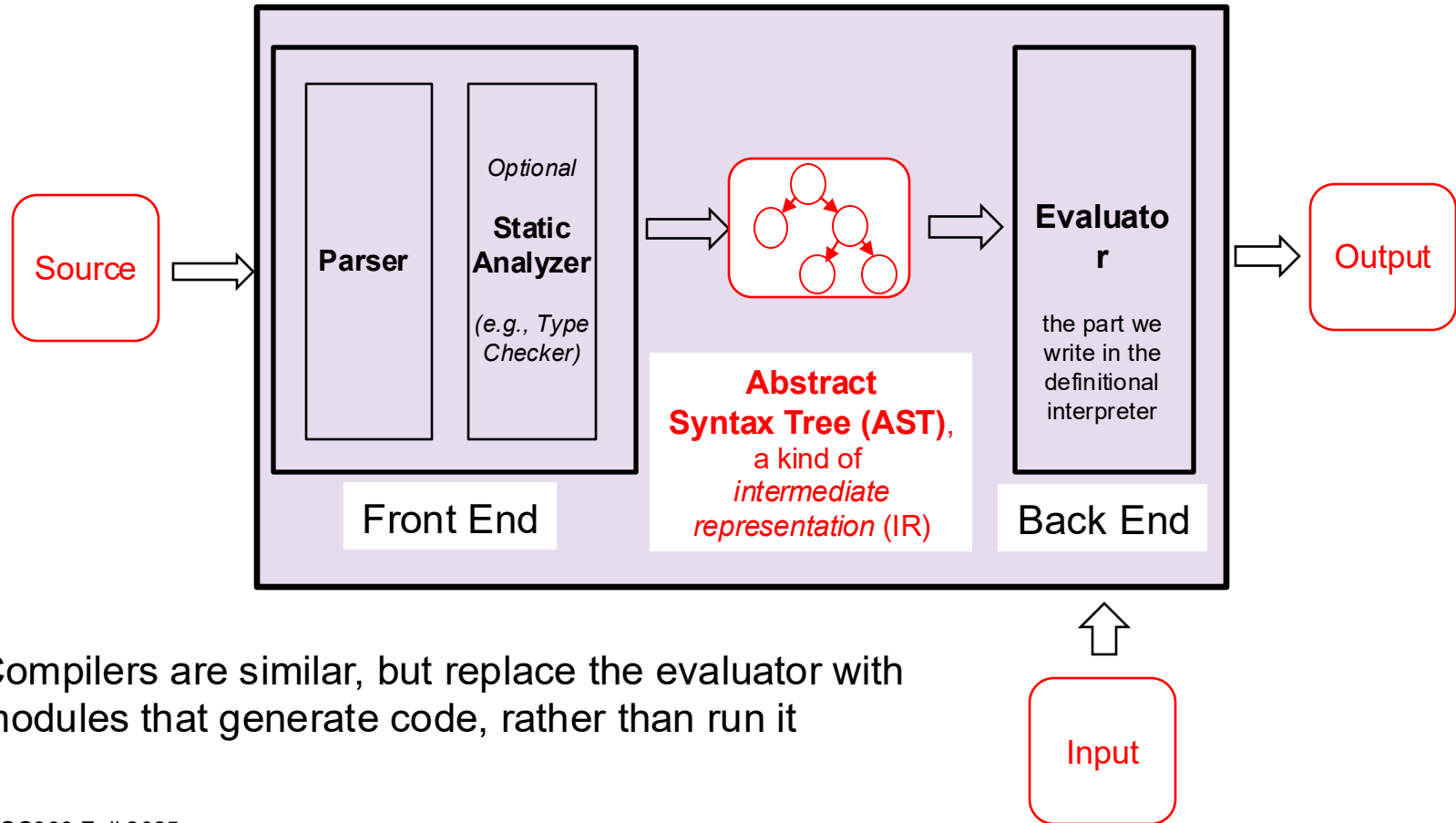


CMSC 330: Organization of Programming Languages

Context Free Grammars

Interpreters

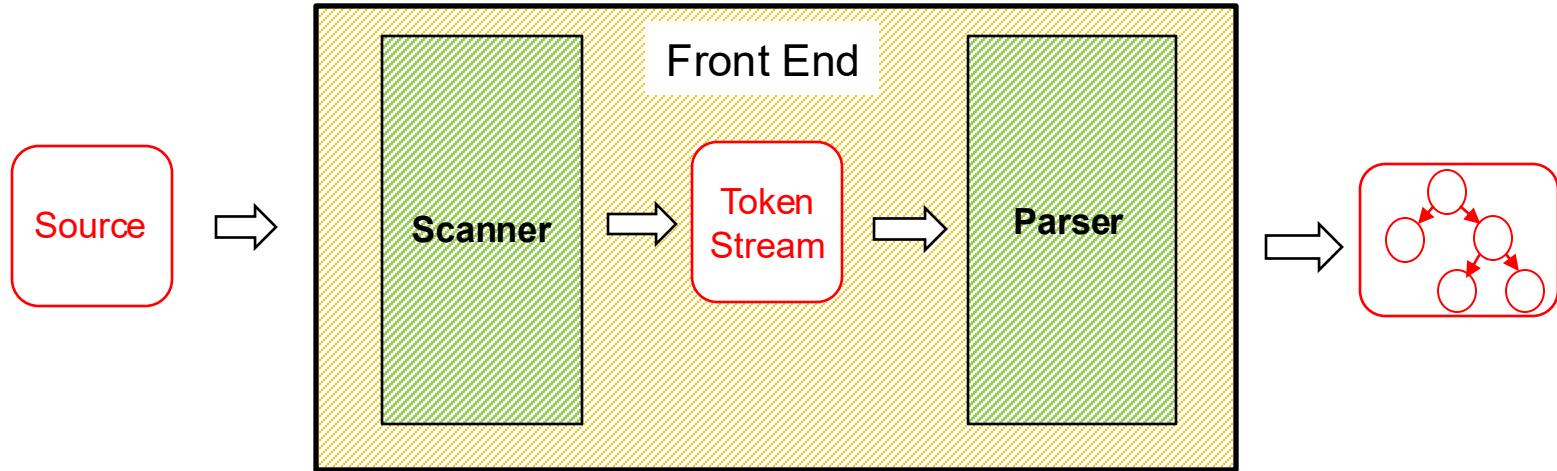


Compilers are similar, but replace the evaluator with modules that generate code, rather than run it

Implementing the Front End

- ▶ Goal: Convert program text into an **Abstract Syntax Tree**
- ▶ ASTs are easier to work with
 - Analyze, optimize, execute the program
- ▶ Do this using regular expressions?
 - **Won't work!**
 - Regular expressions cannot reliably parse paired braces `{ ... }`, parentheses `(( ... ))`, etc.
- ▶ Instead: Regexp for tokens (**scanning**), and **Context Free Grammars** for **parsing** tokens

Front End – Scanner and Parser



- **Scanner / lexer** converts program source into **tokens** (keywords, variable names, operators, numbers, etc.) using **regular expressions**
- **Parser** converts tokens into an **AST** (abstract syntax tree). Parsers recognize strings defined as **context free grammars**

Context-Free Grammar (CFG)

- ▶ A way of describing **sets of strings** (= languages)
 - The notation $L(G)$ denotes the language of strings defined by grammar G
- ▶ Example grammar G is $S \rightarrow 0S \mid 1S \mid \varepsilon$
which says that string $s' \in L(G)$ iff
 - $s' = \varepsilon$, or $\exists s \in L(G)$ such that $s' = 0s$, or $s' = 1s$
- ▶ Grammar is same as regular expression $(0|1)^*$
 - Generates / accepts the same set of strings

CFGs Are Expressive

- ▶ CFGs **subsume** REs, DFAs, NFAs
 - There is a CFG that generates any regular language
 - But: REs are often better notation for those languages
- ▶ And CFGs can define languages regexps cannot
 - $S \rightarrow (S) \mid \epsilon$ // represents balanced pairs of ()'s
- ▶ As a result, CFGs often used as the basis of **parsers** for programming languages

Parsing with CFGs

- ▶ CFGs formally define languages, but they **do not** define an *algorithm* for accepting strings
- ▶ Several styles of algorithm; each works only for less expressive forms of CFG
 - LL(k) parsing  We will discuss this next lecture
 - LR(k) parsing
 - LALR(k) parsing
 - SLR(k) parsing
- ▶ Tools exist for building parsers from grammars
 - JavaCC, Yacc, etc.

Formal Definition: Context-Free Grammar

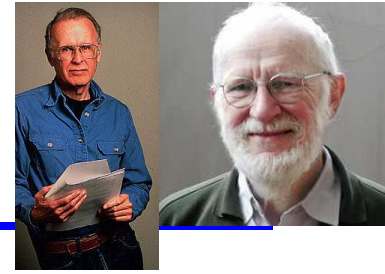
- ▶ A CFG G is a 4-tuple (Σ, N, P, S)
 - Σ – alphabet (finite set of symbols, or terminals)
 - Often written in lowercase
 - N – a finite, nonempty set of nonterminal symbols
 - Often written in UPPERCASE
 - It must be that $N \cap \Sigma = \emptyset$
 - P – a set of productions of the form $N \rightarrow (\Sigma|N)^*$
 - Informally: the nonterminal can be replaced by the string of zero or more terminals / nonterminals to the right of the $\rightarrow$
 - Can think of productions as rewriting rules (more later)
 - $S \in N$ – the start symbol

Notational Shortcuts

$S \rightarrow aBc$	$S \rightarrow aBc$	// S is start symbol
$A \rightarrow aA$		
b		// $A \rightarrow b$
		// $A \rightarrow \varepsilon$

- ▶ A production is of the form
 - left-hand side (LHS) $\rightarrow$ right hand side (RHS)
- ▶ If not specified
 - Assume LHS of first production is the start symbol
- ▶ Productions with the same LHS
 - Are usually combined with |
- ▶ If a production has an empty RHS
 - It means the RHS is ε

Aside: Backus-Naur Form



- ▶ Context-free grammar production rules are also called Backus-Naur Form or **BNF**
 - Designed by John Backus and Peter Naur
 - Chair and Secretary of the Algol committee in the early 1960s. Used this notation to describe Algol in 1962
- ▶ A production is written in BNF as

$$A \rightarrow B \text{ c } D$$
$$\langle A \rangle ::= \langle B \rangle \text{ c } \langle D \rangle$$

Generating Strings

- ▶ Think of a grammar as **generating** strings by **rewriting**
 - Beginning with the start symbol, repeatedly rewrite a nonterminal per a production in the grammar (replace LHS with RHS)

- ▶ Example grammar **G**

$$S \rightarrow 0S \mid 1S \mid \varepsilon$$

- ▶ Generate string 011 from **G** as follows:

$S \Rightarrow 0S$ // using $S \rightarrow 0S$

$\Rightarrow 01S$ // using $S \rightarrow 1S$

$\Rightarrow 011S$ // using $S \rightarrow 1S$

$\Rightarrow 011$ // using $S \rightarrow \varepsilon$

Accepting Strings (Informally)

- ▶ Checking if $s \in L(G)$ is called **acceptance**
 - Algorithm: Find a **rewriting** from G 's start symbol that yields s
 - $011 \in L(G)$ according to the previous rewriting
- ▶ Terminology
 - Such a sequence of rewrites is a **derivation** or **parse**
 - Discovering the derivation is called **parsing**

Derivations

► Notation

- $\Rightarrow$ indicates a derivation of one step
- $\Rightarrow^+$ indicates a derivation of one or more steps
- $\Rightarrow^*$ indicates a derivation of zero or more steps

► Example

- $S \rightarrow 0S \mid 1S \mid \varepsilon$

► For the string 010

- $S \Rightarrow 0S \Rightarrow 01S \Rightarrow 010S \Rightarrow 010$
- $S \Rightarrow^+ 010$
- $010 \Rightarrow^* 010$

Language Generated by Grammar

- ▶ $L(G)$ the language defined by G is

$$L(G) = \{ s \in \Sigma^* \mid S \Rightarrow^+ s \}$$

- S is the start symbol of the grammar
 - Σ is the alphabet for that grammar
- ▶ In other words
 - All strings over Σ that can be derived from the start symbol via one or more productions

Quiz #1

Consider the grammar

$$S \rightarrow bS \mid T$$

$$T \rightarrow aT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$

Which of the following strings is generated by this grammar?

A. aba

B. ccc

C. bab

D. ca

Quiz #1

Consider the grammar

$$S \rightarrow bS \mid T$$

$$T \rightarrow aT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$

Which of the following strings is generated by this grammar?

A. aba

B. ccc

C. bab

D. ca

Quiz #2

- ▶ Consider the grammar

$$S \rightarrow bS \mid T$$

$$T \rightarrow aT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$

- ▶ Which of the following is a derivation of the string **aac**?

A. $S \Rightarrow T \Rightarrow aT \Rightarrow aTaT \Rightarrow aaT \Rightarrow aacU \Rightarrow aac$

B. $S \Rightarrow T \Rightarrow U \Rightarrow aU \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

C. $S \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

D. $S \Rightarrow T \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

Quiz #2

- ▶ Consider the grammar

$$S \rightarrow bS \mid T$$

$$T \rightarrow aT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$

- ▶ Which of the following is a derivation of the string **aac**?

A. $S \Rightarrow T \Rightarrow aT \Rightarrow aTaT \Rightarrow aaT \Rightarrow aacU \Rightarrow aac$

B. $S \Rightarrow T \Rightarrow U \Rightarrow aU \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

C. $S \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

D. $S \Rightarrow T \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

Quiz #3

Consider the grammar

$$S \rightarrow bS \mid T$$

$$T \rightarrow aT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$

Which of the following regular expressions accepts the same language as this grammar?

A. $(a|b|c)^*$

B. $b^*a^*c^*$

C. $(b|ba|bac)^*$

D. bac^*

Quiz #3

Consider the grammar

$$S \rightarrow bS \mid T$$

$$T \rightarrow aT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$

Which of the following regular expressions accepts the same language as this grammar?

A. $(a|b|c)^*$

B. $b^*a^*c^*$

C. $(b|ba|bac)^*$

D. bac^*

Designing Grammars

1. Use recursive productions to generate an arbitrary number of symbols

$A \rightarrow xA \mid \epsilon$ // Zero or more x 's

$A \rightarrow yA \mid y$ // One or more y 's

2. Use separate non-terminals to generate disjoint parts of a language, and then combine in a production

a^*b^* // a 's followed by b 's

$S \rightarrow AB$

$A \rightarrow aA \mid \epsilon$ // Zero or more a 's

$B \rightarrow bB \mid \epsilon$ // Zero or more b 's

Designing Grammars

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

$\{a^n b^n \mid n \geq 0\}$ // N a' s followed by N b' s

$S \rightarrow aSb \mid \epsilon$

Example derivation: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

$\{a^n b^{2n} \mid n \geq 0\}$ // N a' s followed by 2N b' s

$S \rightarrow aSbb \mid \epsilon$

Example derivation: $S \Rightarrow aSbb \Rightarrow aaSbbbb \Rightarrow aabbbb$

Designing Grammars

4. For a language that is the union of other languages, use separate nonterminals for each part of the union and then combine

$$\{ a^n(b^m|c^m) \mid m > n \geq 0 \}$$

Can be rewritten as

$$\{ a^n b^m \mid m > n \geq 0 \} \cup \{ a^n c^m \mid m > n \geq 0 \}$$

$$S \rightarrow T \mid V$$

$$T \rightarrow aTb \mid U$$

$$U \rightarrow Ub \mid b$$

$$V \rightarrow aVc \mid W$$

$$W \rightarrow Wc \mid c$$

Practice

- ▶ Try to make a grammar which accepts

- 0^*1^*
- 0^n1^n where $n \geq 0$

$$S \rightarrow A \mid B$$

$$A \rightarrow 0A \mid \varepsilon$$

$$B \rightarrow 1B \mid \varepsilon$$

$$S \rightarrow 0S1 \mid \varepsilon$$

- ▶ Give some example strings from this language

- $S \rightarrow 0 \mid 1S$

- 0, 10, 110, 1110, 11110, ...

- What language is it, as a regexp?

- 1^*0

Quiz #4

Which of the following grammars describes the same language as $0^n 1^m$ where $m \leq n$?

- A. $S \rightarrow 0S1 \mid \epsilon$
- B. $S \rightarrow 0S1 \mid S1 \mid \epsilon$
- C. $S \rightarrow 0S1 \mid 0S \mid \epsilon$
- D. $S \rightarrow SS \mid 0 \mid 1 \mid \epsilon$

Quiz #4

Which of the following grammars describes the same language as $0^n 1^m$ where $m \leq n$?

A. $S \rightarrow 0S1 \mid \epsilon$

same number of 0 and 1

B. $S \rightarrow 0S1 \mid S1 \mid \epsilon$

more 1's

C. $S \rightarrow 0S1 \mid 0S \mid \epsilon$

more 0's

D. $S \rightarrow SS \mid 0 \mid 1 \mid \epsilon$

no control of the number

Parse Trees

- ▶ Parse tree shows how a string is produced by a grammar
 - *Will be useful for spotting **ambiguity**; discussed later*
 - Root node of parse tree is the start symbol
 - Every internal node is a nonterminal
 - Children of an internal node
 - Are symbols on RHS of production applied to nonterminal
 - Every leaf node is a terminal or ϵ
- ▶ Reading the leaves left to right
 - Shows the string corresponding to the tree

Parse Tree Example

S

S

$S \rightarrow aS \mid T$

$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \varepsilon$

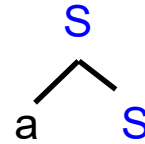
Parse Tree Example

$$S \Rightarrow aS$$

$$S \rightarrow aS \mid T$$

$$T \rightarrow bT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$



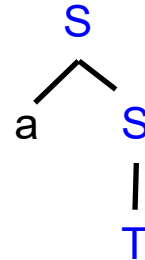
Parse Tree Example

$$S \Rightarrow aS \Rightarrow aT$$

$$S \rightarrow aS \mid T$$

$$T \rightarrow bT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$



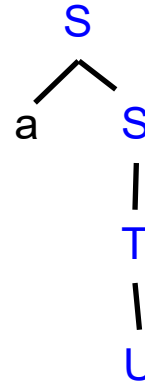
Parse Tree Example

$S \Rightarrow aS \Rightarrow aT \Rightarrow aU$

$S \rightarrow aS \mid T$

$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \varepsilon$



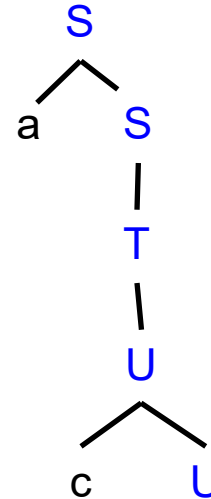
Parse Tree Example

$S \Rightarrow aS \Rightarrow aT \Rightarrow aU \Rightarrow acU$

$S \rightarrow aS \mid T$

$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \varepsilon$



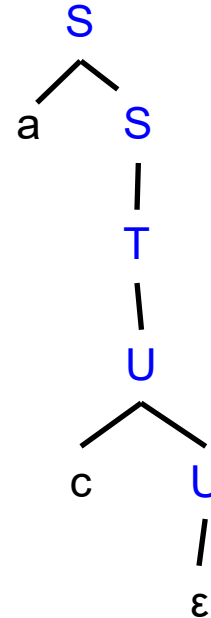
Parse Tree Example

$S \Rightarrow aS \Rightarrow aT \Rightarrow aU \Rightarrow acU \Rightarrow ac$

$S \rightarrow aS \mid T$

$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \varepsilon$



CFGs and ASTs

- ▶ An **abstract syntax tree** is a data structure that represents a parsed input, e.g., a program expression
 - An AST can be expressed with an OCaml datatype that is very close to the CFG that describes the language syntax

CFG for arithmetic expressions:

▶ $E \rightarrow a \mid b \mid c \mid d$
| $E + E$
| $E - E$
| $E * E$
| (E)

AST:

```
type expr = A | B | C | D
          | Plus of expr * expr
          | Minus of expr * expr
          | Mult of expr * expr
```

Eventual Goal: Parse a CFG to get an AST

CFG (string):

► $E \rightarrow a \mid b \mid c \mid d$
| $E + E$
| $E - E$
| $E * E$
| (E)

AST definition (OCaml):

```
type expr = A | B | C | D  
| Plus of expr * expr  
| Minus of expr * expr  
| Mult of expr * expr
```

a-c parses to

a-(b*a) parses to

c*(b+d) parses to

Minus (A, C)

Minus (A, Mult (B,A))

Mult (C, Plus (B,D))

Parse Trees for Expressions

- ▶ A **parse tree** shows the structure of an expression as it corresponds to a grammar

$$E \rightarrow a \mid b \mid c \mid d \mid E+E \mid E-E \mid E^*E \mid (E)$$

a

a*c

c*(b+d)

Parse Trees for Expressions

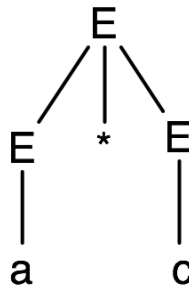
- ▶ A **parse tree** shows the structure of an expression as it corresponds to a grammar

$$E \rightarrow a \mid b \mid c \mid d \mid E+E \mid E-E \mid E^*E \mid (E)$$

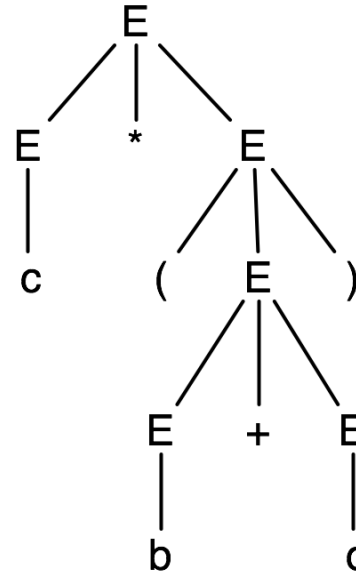
a



a*c



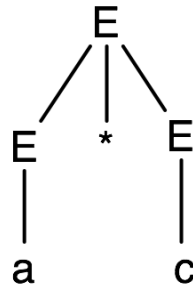
c*(b+d)



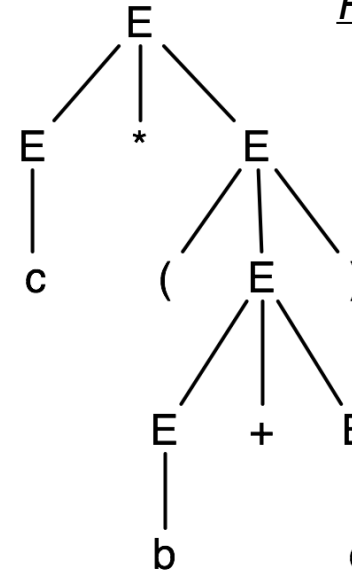
Abstract Syntax Trees

- ▶ A **parse tree** and an **AST** are **not the same thing**
 - The latter is a data structure produced by parsing

$a * c$

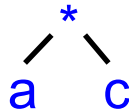


$c * (b + d)$

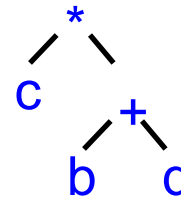


Parse trees

ASTs



`Mult(A, C)`



`Mult(C, Plus(B, D))`

Practice

$E \rightarrow a \mid b \mid c \mid d \mid E+E \mid E-E \mid E^*E \mid (E)$

Make a parse tree for...

- a^*b
- $a+(b-c)$
- $d^*(d+b)-a$
- $(a+b)^*(c-d)$
- $a+(b-c)^*d$

Leftmost and Rightmost Derivation

- ▶ Leftmost derivation
 - Leftmost nonterminal is replaced in each step
- ▶ Rightmost derivation
 - Rightmost nonterminal is replaced in each step
- ▶ Example
 - Grammar
 - $S \rightarrow AB, A \rightarrow a, B \rightarrow b$
 - Leftmost derivation for “ab”
 - $S \Rightarrow AB \Rightarrow aB \Rightarrow ab$
 - Rightmost derivation for “ab”
 - $S \Rightarrow AB \Rightarrow Ab \Rightarrow ab$

Parse Tree For Derivations

- ▶ Parse tree may be same for both leftmost & rightmost derivations

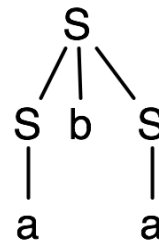
- Example Grammar: $S \rightarrow a \mid SbS$ String: aba

Leftmost Derivation

$S \Rightarrow SbS \Rightarrow abS \Rightarrow aba$

Rightmost Derivation

$S \Rightarrow SbS \Rightarrow Sba \Rightarrow aba$



- Parse trees don't show order productions are applied
- Every parse tree has a unique leftmost and a unique rightmost derivation

Parse Tree For Derivations (cont.)

- ▶ Not every string has a unique parse tree

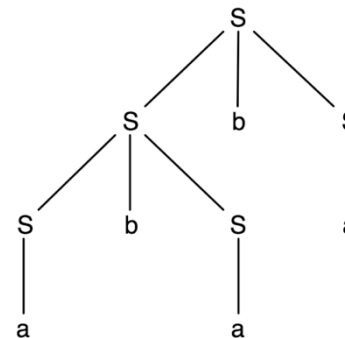
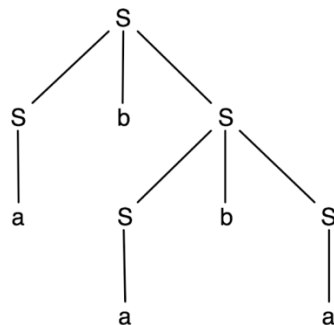
- Example Grammar: $S \rightarrow a \mid SbS$ String: **ababa**

Leftmost Derivation

$S \Rightarrow SbS \Rightarrow abS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$

Another Leftmost Derivation

$S \Rightarrow SbS \Rightarrow SbSbS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$



Ambiguity

- ▶ A grammar is **ambiguous** if a string may have multiple **leftmost** derivations

I saw a girl with a telescope.



Ambiguity

- ▶ A grammar is **ambiguous** if a string may have multiple **leftmost** derivations

- Equivalent to multiple parse trees
- Can be hard to determine

1. $S \rightarrow aS \mid T$

$$T \rightarrow bT \mid U$$

$$U \rightarrow cU \mid \varepsilon$$

No

2. $S \rightarrow T \mid T$

$$T \rightarrow Tx \mid Tx \mid x \mid x$$

Yes

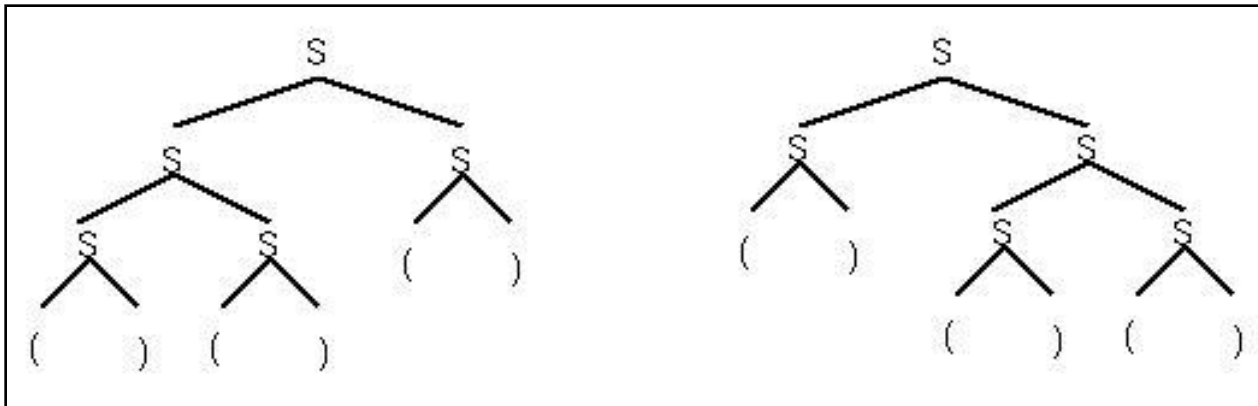
3. $S \rightarrow SS \mid () \mid (S)$

?

Ambiguity (cont.)

► Example

- Grammar: $S \rightarrow SS \mid () \mid (S)$ String: $()()()$
- 2 distinct (leftmost) derivations (and parse trees)
 - $S \Rightarrow \underline{SS} \Rightarrow \underline{SSS} \Rightarrow ()\underline{SS} \Rightarrow ()()\underline{S} \Rightarrow ()()()$
 - $S \Rightarrow \underline{SS} \Rightarrow ()\underline{S} \Rightarrow ()\underline{SS} \Rightarrow ()()\underline{S} \Rightarrow ()()()$

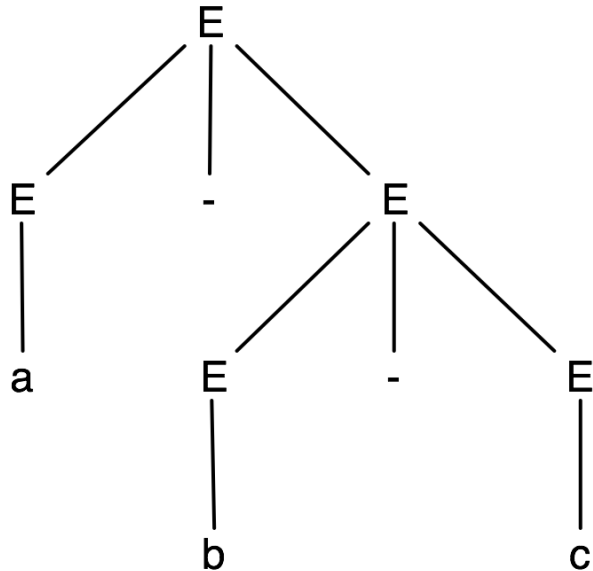


CFGs for Programming Languages

- ▶ Recall that our goal is to describe programming languages with CFGs
- ▶ We had the following example which describes limited arithmetic expressions
$$E \rightarrow a \mid b \mid c \mid E+E \mid E-E \mid E^*E \mid (E)$$
- ▶ What's wrong with using this grammar?
 - It's ambiguous!

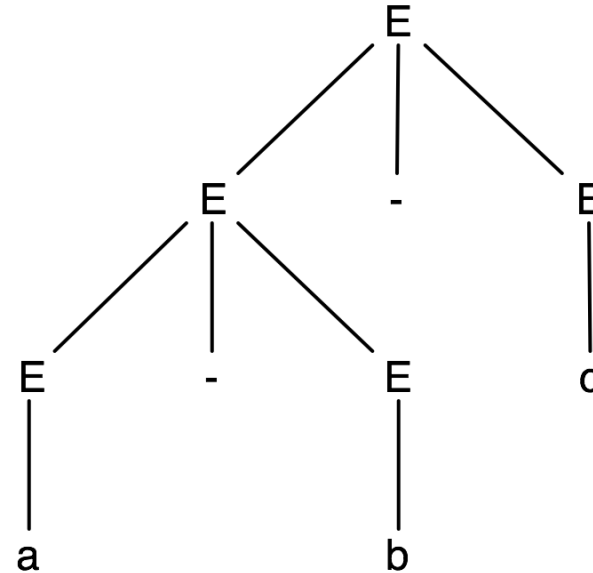
Example: a-b-c

$E \Rightarrow E-E \Rightarrow a-E \Rightarrow a-E-E \Rightarrow$
 $a-b-E \Rightarrow a-b-c$



Corresponds to $a-(b-c)$

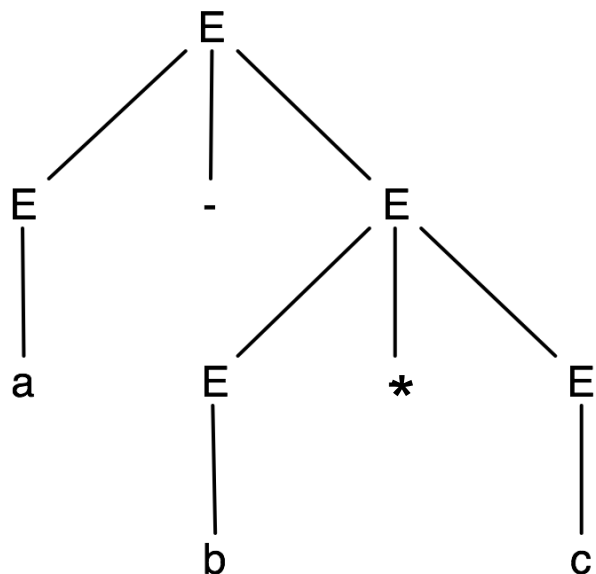
$E \Rightarrow E-E \Rightarrow E-E-E \Rightarrow$
 $a-E-E \Rightarrow a-b-E \Rightarrow a-b-c$



Corresponds to $(a-b)-c$

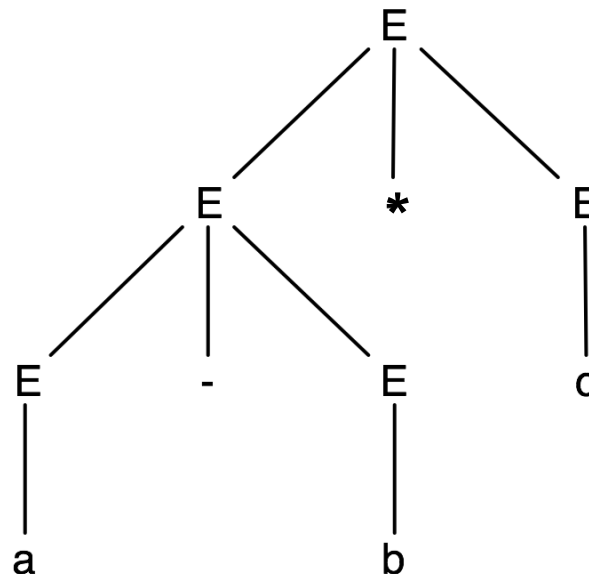
Example: $a-b^*c$

$E \Rightarrow E-E \Rightarrow a-E \Rightarrow a-E^*E \Rightarrow$
 $a-b^*E \Rightarrow a-b^*c$



Corresponds to $a-(b^*c)$

$E \Rightarrow E-E \Rightarrow E-E^*E \Rightarrow$
 $a-E^*E \Rightarrow a-b^*E \Rightarrow a-b^*c$



Corresponds to $(a-b)^*c$

Another Example: If-Then-Else

Aka the dangling else problem

$\langle \text{stmt} \rangle \rightarrow \langle \text{assignment} \rangle \mid \langle \text{if-stmt} \rangle \mid \dots$

$\langle \text{if-stmt} \rangle \rightarrow \text{if } (\langle \text{expr} \rangle) \langle \text{stmt} \rangle \mid$
 $\quad \text{if } (\langle \text{expr} \rangle) \langle \text{stmt} \rangle \text{ else } \langle \text{stmt} \rangle$

(Recall $\langle \rangle$'s are used to denote nonterminals)

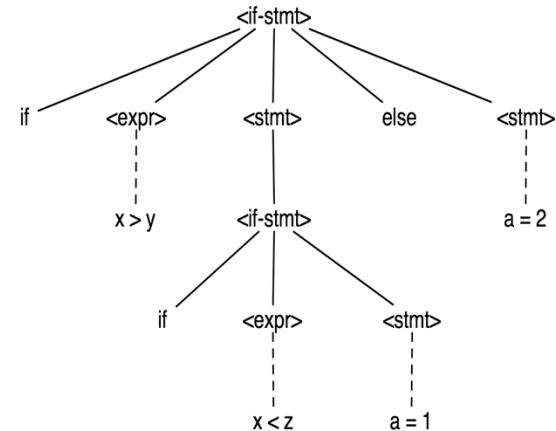
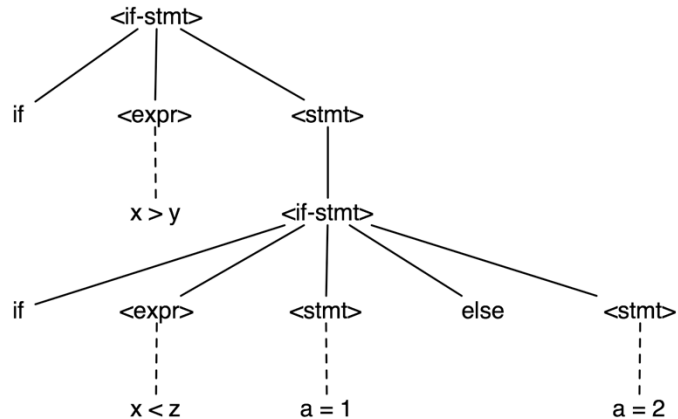
- Consider the following program fragment

```
if (x > y)
  if (x < z)
    a = 1;
  else a = 2;
```

(Note: Ignore newlines)

Two Parse Trees

```
if (x > y)
    if (x < z)
        a = 1;
    else a = 2;
```



Quiz #5

Which of the following grammars is ambiguous?

A. $S \rightarrow 0SS1 \mid 0S1 \mid \varepsilon$

B. $S \rightarrow A1S1A \mid \varepsilon$

$$A \rightarrow 0$$

C. $S \rightarrow (S, S, S) \mid 1$

D. None of the above.

Quiz #5

Which of the following grammars is ambiguous?

A. $S \rightarrow 0SS1 \mid 0S1 \mid \epsilon$

B. $S \rightarrow A1S1A \mid \epsilon$

$A \rightarrow 0$

C. $S \rightarrow (S, S, S) \mid 1$

D. None of the above.

$S \rightarrow 0SS1 \rightarrow 0S1 \rightarrow 01$
 $S \rightarrow 0S1 \rightarrow 01$

Dealing With Ambiguous Grammars

- ▶ Ambiguity is bad
 - Syntax is correct
 - But semantics differ depending on choice
 - Different associativity $(a-b)-c$ vs. $a-(b-c)$
 - Different precedence $(a-b)*c$ vs. $a-(b*c)$
 - Different control flow if (if else) vs. if (if) else
- ▶ Two approaches
 - Rewrite grammar
 - **Grammars are not unique** – can have multiple grammars for the same language. But result in different parses.
 - Use special parsing rules
 - Depending on parsing tool

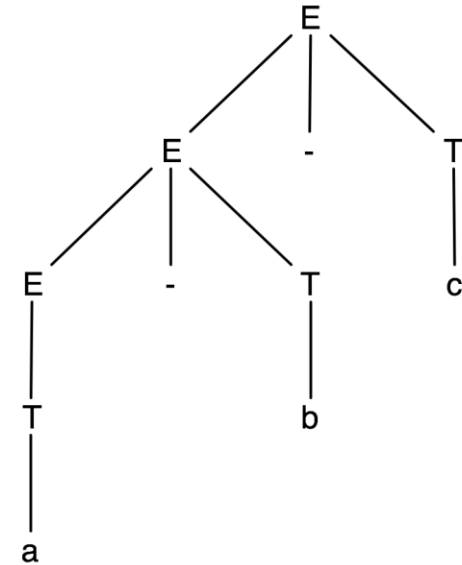
Fixing the Expression Grammar

- Require right operand to not be bare expression

$$E \rightarrow E+T \mid E-T \mid E*T \mid T$$

$$T \rightarrow a \mid b \mid c \mid (E)$$

- Corresponds to **left associativity**
- Now only one parse tree for **a-b-c**
 - Find derivation



What if we want Right Associativity?

- ▶ Left-recursive productions

- Used for left-associative operators
- Example

$$E \rightarrow E+T \mid E-T \mid E*T \mid T$$
$$T \rightarrow a \mid b \mid c \mid (E)$$

- ▶ Right-recursive productions

- Used for right-associative operators
- Example

$$E \rightarrow T+E \mid T-E \mid T^*E \mid T$$
$$T \rightarrow a \mid b \mid c \mid (E)$$

A Different Problem

- ▶ How about the string $a+b*c$?

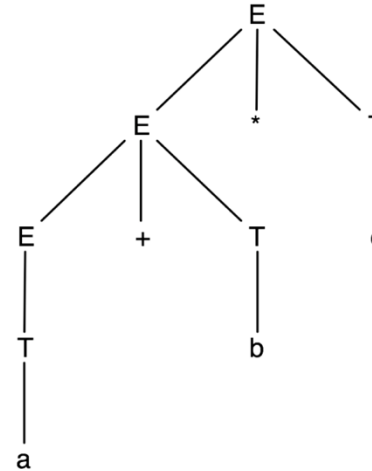
$E \rightarrow E+T \mid E-T \mid E*T \mid T$

$T \rightarrow a \mid b \mid c \mid (E)$

- ▶ Doesn't have correct precedence for $*$

- When a nonterminal has productions for several operators, they effectively have the same precedence

- ▶ Solution – Introduce **new** nonterminals



Final Expression Grammar

$E \rightarrow E+T \mid E-T \mid T$	lowest precedence operators
$T \rightarrow T*P \mid P$	higher precedence
$P \rightarrow a \mid b \mid c \mid (E)$	highest precedence (parentheses)

Derivation of $a+b*c$:

$$E \rightarrow E+T \rightarrow T+T \rightarrow P+T \rightarrow a+T \rightarrow a+T*P \rightarrow a+P*P \rightarrow a+b*P \rightarrow a+b*c$$

Fixing the Expression Grammar

- ▶ Controlling precedence of operators
 - Introduce new nonterminals
 - Precedence increases closer to operands
- ▶ Controlling associativity of operators
 - Introduce new nonterminals
 - Assign associativity based on production form
 - $E \rightarrow E+T$ (left associative) vs. $E \rightarrow T+E$ (right associative)
 - But parsing method might limit form of rules

Conclusion

- ▶ Context Free Grammars (CFGs) can describe programming language syntax
 - They are a kind of formal language that is more powerful than regular expressions
- ▶ CFGs can also be used as the basis for programming language parsers (details later)
 - But the grammar should not be ambiguous
 - May need to change more natural grammar to make it so
 - Parsing often aims to produce abstract syntax trees
 - Data structure that records the key elements of program