CMSC 330: Organization of Programming Languages

Tail Recursion

Factorial

$$\text{fact } n = \begin{cases} 1 & n=0 \\ n * \text{fact } (n-1) & n>0 \end{cases}$$

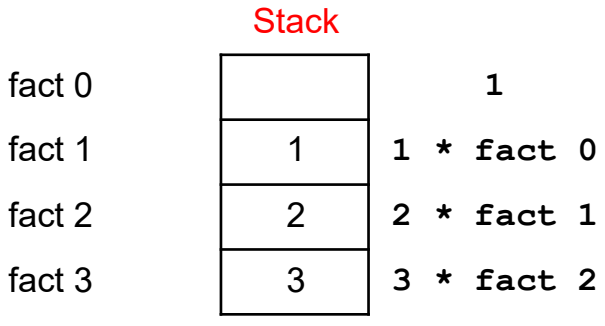
```
let rec fact  n =
    if n = 0 then 1
    else n * fact (n-1)
```

```
fact 4 = 24
```

Factorial

$$\text{fact } n = \begin{cases} 1 & n=0 \\ n * \text{fact } (n-1) & n>0 \end{cases}$$

```
fact 3 = 3 * fact 2
       = 3 * 2 * fact 1
       = 3 * 2 * 1 * fact 0
       = 3 * 2 * 1 * 1
       = 3 * 2 * 1
       = 3 * 2
       = 6
```



Stack Overflow

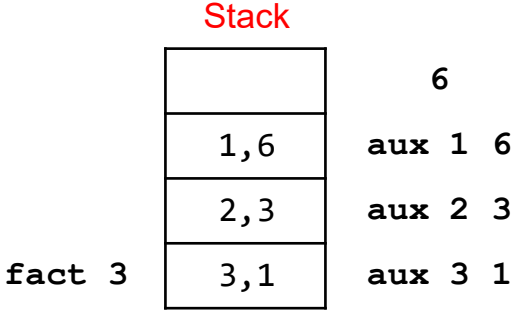
```
# let rec fact n = if n = 0 then 1 else n * fact (n-1);;
val fact : int -> int = <fun>
# fact 1000000 ;;
Stack overflow during evaluation (looping recursion?).
```

Yet Another Factorial

$$\texttt{aux x a} = \begin{cases} \texttt{a} & \texttt{x=0} \\ \texttt{aux (x-1) x*a} & \texttt{x>0} \end{cases}$$

```
fact n  = aux n 1
```

```
let fact n =
  let rec aux x a =
    if x = 0 then a
    else aux (x-1) x*a
  in
  aux n 1
```



Yet Another Factorial

```
aux x a = { a          x=0
          { aux (x-1) x*a   x>0

fact n  = aux n 1
```

```
fact 3 = aux 3 1
       = aux 2 3
       = aux 1 6
       = 6
```

Tail Recursion

- Whenever a function's result is completely computed by its recursive call, it is called **tail recursive**
  - Its "tail" – the last thing it does – is recursive
- Tail recursive functions can be implemented without requiring a stack frame for each call
  - No intermediate variables need to be saved, so the compiler overwrites them
- Typical pattern is to use an accumulator to build up the result, and return it in the base case

Compare fact and aux

```
let rec fact n =
    if n = 0 then 1
    else n * fact (n-1)
```

Waits for recursive call's result to compute final result

```
let fact n =
  let rec aux x acc =
    if x = 1 then acc
    else aux (x-1) (acc*x)
  in
 aux n 1
```

*final result **is** the result of the recursive call*

Exercise: Finish Tail-recursive Version

```
let rec sumlist l =
  match l with
    [] -> 0
  | (x::xs) -> (sumlist xs) + x
```

Tail-recursive version:

```
let sumlist l =
  let rec helper l a =
    match l with
      [] -> a
    | (x::xs) -> helper xs (x+a)
  in
helper l 0
```

A Tail Recursive **map**

```
let map f l =
  let rec helper l a =
    match l with
      [] -> a
    | h::t -> helper t ((f h)::a)
  in rev (helper l [])
```

Could instead change `(f h)::a` to be `a@(f h)`

Q: Why is the above implementation a better choice?

A: $O(n)$ running time, not $O(n^2)$ (where n is length of list)