Discrete Structures

Rules of inference and Arguments
9/8/2022

Review

Review

Conditional Statement

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Review

Conditional Statement

$p \Rightarrow q \equiv \neg(p \wedge \neg q) \equiv (\neg p \vee q)$

p	q	$p \Rightarrow q$	$\neg(p \wedge \neg q)$	$(\neg p \vee q)$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	1	1

Review

If I am in IRB, then I am on campus.

$p \Rightarrow q$: I am in IRB ⇒ I am on campus.

$q \Rightarrow p$
Converse: I am on campus ⇒ I am in IRB

$\neg p \Rightarrow \neg q$
Inverse: If I am not in IRB then I am not on campus.

$\neg q \Rightarrow \neg p$
Contrapositive: if I am not on campus then I am not in IRB

Conditionals

Sufficient
r is a **sufficient condition** for s means "if r then s."

$r \Rightarrow s$

Necessary
r is a **necessary condition** for s means "if not r then not s."

$\neg r \Rightarrow \neg s$

$s \Rightarrow r$

What does it mean if r is **necessary** and **sufficient** for s?

Laws of Equivalence

1. Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity	$p \wedge 1 \equiv p$	$p \vee 0 \equiv p$
5. Negation	$p \vee \neg p \equiv 1$	$p \wedge \neg p \equiv 0$
6. Double negative	$\neg(\neg p) \equiv p$	
7. Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound	$p \vee 1 \equiv 1$	$p \wedge 0 \equiv 0$
9. De Morgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
10. Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of t and c	$\neg 1 \equiv 0$	$\neg 0 \equiv 1$

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

$p \land \neg q \lor r$

$\equiv$

Precedence Clarification

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

$p \land \neg q \lor r$

$\equiv \quad (p \land \neg q) \lor r$ Precedence Clarification

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$		Double Negative Law

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law

De Morgan’s Law

Apply De Morgan’s Law to:
$p \wedge \neg q \vee r \equiv \neg((\neg p \vee q) \wedge \neg r)$

	$p \wedge \neg q \vee r$	
$\equiv$	$(p \wedge \neg q) \vee r$	Precedence Clarification
$\equiv$	$\neg\neg((p \wedge \neg q) \vee r)$	Double Negative Law
$\equiv$		De Morgan’s Law

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$		

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$		De Morgan's Law
$\equiv$		

De Morgan’s Law

Apply De Morgan’s Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan’s Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan’s Law
$\equiv$		

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan's Law
$\equiv$		

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan's Law
$\equiv$		Double Negative Law

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor q) \land \neg r)$	Double Negative Law

Practice

Prove

$(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv \neg q$

$(p \vee \neg q) \wedge (\neg p \vee \neg q)$

$\equiv$

Practice

Prove

$(p \lor \neg q) \land (\neg p \lor \neg q) \equiv \neg q$

$(p \lor \neg q) \land (\neg p \lor \neg q)$

$\equiv (\neg q \lor p) \land (\neg q \lor \neg p)$ Commutative Law

Practice

Prove
$(p \lor \neg q) \land (\neg p \lor \neg q) \equiv \neg q$

	$(p \lor \neg q) \land (\neg p \lor \neg q)$	
$\equiv$	$(\neg q \lor p) \land (\neg q \lor \neg p)$	Commutative Law
$\equiv$	$\neg q \lor (p \land \neg p)$	Distributive Law

Practice

Prove
$(p \lor \neg q) \land (\neg p \lor \neg q) \equiv \neg q$

	$(p \lor \neg q) \land (\neg p \lor \neg q)$	
$\equiv$	$(\neg q \lor p) \land (\neg q \lor \neg p)$	Commutative Law
$\equiv$	$\neg q \lor (p \land \neg p)$	Distributive Law
$\equiv$	$\neg q \lor 0$	Negation Law

Practice

Prove

$(p \lor \neg q) \land (\neg p \lor \neg q) \equiv \neg q$

	$(p \lor \neg q) \land (\neg p \lor \neg q)$	
$\equiv$	$(\neg q \lor p) \land (\neg q \lor \neg p)$	Commutative Law
$\equiv$	$\neg q \lor (p \land \neg p)$	Distributive Law
$\equiv$	$\neg q \lor 0$	Negation Law
$\equiv$	$\neg q$	Identity Law

Conditionals

Sufficient
r is a **sufficient condition** for s means “if r then s.”

$r \Rightarrow s$

Necessary
r is a **necessary condition** for s means “if not r then not s.”

$\neg r \Rightarrow \neg s$

$s \Rightarrow r$

What does it mean if r is **necessary** and **sufficient** for s?

Arguments & Argument Forms

Arguments

Sequence of statements ending in a conclusion.

In a **valid argument form,** the conclusion follows *necessarily* from the preceding statements.

If an argument is valid, it has a valid form.

Argument

If Socrates is a man, then Socrates is mortal.

Socrates is a man.

∴ Socrates is mortal

Argument *Form*

If p, then q.

p

∴ q

Argument

Consistency check/style guide

(1)	premise one	
(2)	premise two	
(3)	argument 1	justification
(4)	argument 2	justification
	⋮	
∴	*conclusion*	justification

Argument

An argument is **valid** if, and only if, whenever statements are substituted that make all the premises true, the conclusion is also true.

Determining Validity of an Argument Form

$p \Rightarrow q \lor \neg r$

$q \Rightarrow p \land r$

$\therefore p \Rightarrow r$

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0						
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0						
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1					
0	0	1	0					
0	1	0	1					
0	1	1	0					
1	0	0	1					
1	0	1	0					
1	1	0	1					
1	1	1	0					

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1					
0	0	1	0					
0	1	0	1					
0	1	1	0					
1	0	0	1					
1	0	1	0					
1	1	0	1					
1	1	1	0					

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1				
0	0	1	0	0				
0	1	0	1	1				
0	1	1	0	1				
1	0	0	1	1				
1	0	1	0	0				
1	1	0	1	1				
1	1	1	0	1				

Determining Validity of an Argument Form

$p \Rightarrow q \lor \neg r$

$q \Rightarrow p \land r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \lor \neg r$	$p \land r$	$p \Rightarrow q \lor \neg r$	$q \Rightarrow p \land r$	$p \Rightarrow r$
0	0	0	1	1				
0	0	1	0	0				
0	1	0	1	1				
0	1	1	0	1				
1	0	0	1	1				
1	0	1	0	0				
1	1	0	1	1				
1	1	1	0	1				

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0			
0	0	1	0	0	0			
0	1	0	1	1	0			
0	1	1	0	1	0			
1	0	0	1	1	0			
1	0	1	0	0	1			
1	1	0	1	1	0			
1	1	1	0	1	1			

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0			
0	0	1	0	0	0			
0	1	0	1	1	0			
0	1	1	0	1	0			
1	0	0	1	1	0			
1	0	1	0	0	1			
1	1	0	1	1	0			
1	1	1	0	1	1			

Determining Validity of an Argument Form

$p \Rightarrow q \lor \neg r$

$q \Rightarrow p \land r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \lor \neg r$	$p \land r$	$p \Rightarrow q \lor \neg r$	$q \Rightarrow p \land r$	$p \Rightarrow r$
0	0	0	1	1	0	1		
0	0	1	0	0	0	1		
0	1	0	1	1	0	1		
0	1	1	0	1	0	1		
1	0	0	1	1	0			
1	0	1	0	0	1			
1	1	0	1	1	0			
1	1	1	0	1	1			

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0	1		
0	0	1	0	0	0	1		
0	1	0	1	1	0	1		
0	1	1	0	1	0	1		
1	0	0	1	1	0	1		
1	0	1	0	0	1	0		
1	1	0	1	1	0	1		
1	1	1	0	1	1	1		

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0	1		
0	0	1	0	0	0	1		
0	1	0	1	1	0	1		
0	1	1	0	1	0	1		
1	0	0	1	1	0	1		
1	0	1	0	0	1	0		
1	1	0	1	1	0	1		
1	1	1	0	1	1	1		

Determining Validity of an Argument Form

$p \Rightarrow q \lor \neg r$

$q \Rightarrow p \land r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \lor \neg r$	$p \land r$	$p \Rightarrow q \lor \neg r$	$q \Rightarrow p \land r$	$p \Rightarrow r$
0	0	0	1	1	0	1	1	
0	0	1	0	0	0	1	1	
0	1	0	1	1	0	1		
0	1	1	0	1	0	1		
1	0	0	1	1	0	1	1	
1	0	1	0	0	1	0	1	
1	1	0	1	1	0	1		
1	1	1	0	1	1	1		

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0	1	1	
0	0	1	0	0	0	1	1	
0	1	0	1	1	0	1	0	
0	1	1	0	1	0	1	0	
1	0	0	1	1	0	1	1	
1	0	1	0	0	1	0	1	
1	1	0	1	1	0	1	0	
1	1	1	0	1	1	1	1	

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0	1	1	
0	0	1	0	0	0	1	1	
0	1	0	1	1	0	1	0	
0	1	1	0	1	0	1	0	
1	0	0	1	1	0	1	1	
1	0	1	0	0	1	0	1	
1	1	0	1	1	0	1	0	
1	1	1	0	1	1	1	1	

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0	1	1	1
0	0	1	0	0	0	1	1	1
0	1	0	1	1	0	1	0	1
0	1	1	0	1	0	1	0	1
1	0	0	1	1	0	1	1	0
1	0	1	0	0	1	0	1	1
1	1	0	1	1	0	1	0	0
1	1	1	0	1	1	1	1	1

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0	1	1	1
0	0	1	0	0	0	1	1	1
0	1	0	1	1	0	1	0	1
0	1	1	0	1	0	1	0	1
1	0	0	1	1	0	1	1	0
1	0	1	0	0	1	0	1	1
1	1	0	1	1	0	1	0	0
1	1	1	0	1	1	1	1	1

Determining Validity of an Argument Form

$p \Rightarrow q \vee \neg r$

$q \Rightarrow p \wedge r$

$\therefore p \Rightarrow r$

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge r$	$p \Rightarrow q \vee \neg r$	$q \Rightarrow p \wedge r$	$p \Rightarrow r$
0	0	0	1	1	0	1	1	1
0	0	1	0	0	0	1	1	1
0	1	0	1	1	0	1	0	1
0	1	1	0	1	0	1	0	1
1	0	0	1	1	0	1	1	0
1	0	1	0	0	1	0	1	1
1	1	0	1	1	0	1	0	0
1	1	1	0	1	1	1	1	1

INVALID ARGUMENT FORM

The Laws of Inference

Laws of Inference

Modus ponens

If p then q.

p

∴ q

Laws of Inference

Modus ponens

If p then q.

p

∴ q

Validity Check:

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Laws of Inference

Modus ponens

If p then q.

p

∴ q

Validity Check:

p	q	$p \Rightarrow q$
0	0	
0	1	
1	0	
1	1	

Laws of Inference

Modus ponens

If p then q.

p

∴ q

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Laws of Inference

Modus ponens

If p then q.

p

∴ q

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

$p \Rightarrow q$	p	q
1	0	0
1	0	1
0	1	0
1	1	1

Laws of Inference

Modus ponens

If p then q.

p

∴ q

VALID ARGUMENT FORM

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

$p \Rightarrow q$	p	q
1	0	0
1	0	1
0	1	0
1	1	1

Laws of Inference

Modus tollens

If p then q.

$\neg q$

$\therefore \neg p$

Laws of Inference

Modus tollens

If it is raining, then it is cloudy.
It is not cloudy.

Therefore, it is not raining.

If p then q.

$\neg q$

$\therefore \neg p$

Laws of Inference

Modus tollens

If Zeus is human, then Zeus is mortal.

Zeus is not mortal.

∴ Zeus is not human.

If p then q.

$\neg q$

$\therefore \neg p$

Modus tollens

Warning:

Studies by cognitive psychologists have shown that although nearly 100% of college students have a solid, intuitive understanding of modus ponens, less than 60% are able to apply modus tollens correctly.

Practice: modus ponens or modus tollens

If there are more pigeons than there are pigeonholes, then at least two pigeons roost in the same hole.

There are more pigeons than there are pigeonholes.

∴

Practice: modus ponens or modus tollens

If there are more pigeons than there are pigeonholes, then at least two pigeons roost in the same hole.

There are more pigeons than there are pigeonholes.

∴ At least two pigeons roost in the same hole. by modus ponens

∴

Practice: modus ponens or modus tollens

If 870,232 is divisible by 6, then it is divisible by 3.

870,232 is not divisible by 3.

∴

∴

Practice: modus ponens or modus tollens

If 870,232 is divisible by 6, then it is divisible by 3.

870,232 is not divisible by 3.

∴ 870,232 is not divisible by 6. by modus tollens

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization

Generalization

$$\frac{p}{\therefore p \lor q} \quad \text{Generalization}$$

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization
- Specialization

Specialization

$$\frac{p \land q}{\therefore p} \quad \text{Specialization}$$

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization
- Specialization
- Conjunction

Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$$ Conjunction

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization
- Specialization
- Conjunction
- Elimination

Elimination

$p \lor q$

$\neg p$

$\therefore q$ Elimination

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization
- Specialization
- Conjunction
- Elimination
- Transitivity

Transitivity

$$\begin{array}{l} p \Rightarrow q \\ q \Rightarrow r \\ \hline \therefore p \Rightarrow r \end{array}$$

Transitivity

Transitivity

If it is raining, then it is cloudy.
If it cloudy, then there is shade.

∴ If it is raining, then there is shade.

$p \Rightarrow q$

$q \Rightarrow r$

$\therefore p \Rightarrow r$ Transitivity

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization
- Specialization
- Conjunction
- Elimination
- Transitivity
- Cases

Cases

If it is raining or it is snowing.
If it is raining, then it is cloudy.
If it is snowing, then it is cloudy.

∴ It is cloudy.

$p \vee q$

$p \Rightarrow r$

$q \Rightarrow r$

$\therefore r$ Cases

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization
- Specialization
- Conjunction
- Elimination
- Transitivity
- Cases
- Contradiction

Contradiction

$$\frac{\neg p \Rightarrow 0}{\therefore p} \quad \text{Contradiction}$$

Contradiction

$$\frac{\neg p \Rightarrow 0}{\therefore p} \quad \text{Contradiction}$$

Informal Example:
A prime number has exactly 2 factors.
7 is prime.
1 * 1 * 7 = 7
If 1 is prime, then 1 * 1 * 7 is a prime factorization of 7.

The conclusion is FALSE (disagrees with definition of prime numbers having only 2 factors.
∴ 1 is NOT prime.

Contradiction

$$\frac{\neg p \Rightarrow 0}{\therefore p} \quad \text{Contradiction}$$

Recall $s \Rightarrow t \equiv \neg(s \wedge \neg t)$

$s = \neg p$

$t = 0$

$\neg(\neg p \wedge \neg 0)$	*Substitution*
$\neg(\neg p \wedge 1)$	*Negation of c*
$(\neg\neg p \vee \neg 1)$	*De Morgan's Law*
$(\neg\neg p \vee 0)$	*Negation of t*
$(p \vee 0)$	*Double Negative*
p	*Identity*

Rules of Inference

- Modus ponens
- Modus tollens
- Generalization
- Specialization
- Conjunction
- Elimination
- Transitivity
- Cases
- Contradiction
- Dilema

Dilema

$(p \Rightarrow q) \wedge (r \Rightarrow s)$

$(p \vee r)$

$\therefore (q \vee s)$ Dilema

Modus Ponens	Modus Tollens	Generalization
$p \Rightarrow q$ p $\therefore \quad q$	$p \Rightarrow q$ $\sim q$ $\therefore \quad \sim p$	p $\therefore \quad p \vee q$
Specialization	**Conjunction**	**Elimination**
$p \wedge q$ $\therefore \quad p$	p q $\therefore \quad p \wedge q$	$p \vee q$ $\sim p$ $\therefore \quad q$
Transitivity	**Cases**	**Contradiction**
$p \Rightarrow q$ $q \Rightarrow r$ $\therefore \quad p \Rightarrow r$	$p \vee q$ $p \Rightarrow r$ $q \Rightarrow r$ $\therefore \quad r$	$\sim p \Rightarrow 0$ $\therefore \quad p$
Dilema		
$(p \Rightarrow q) \wedge (r \Rightarrow s)$ $(p \vee r)$ $\therefore \quad (q \vee s)$		

Argument Valid/Invalid

If Zeke is a cheater, then Zeke sits in the back row.

Zeke sits in the back row.

∴ Zeke is a cheater.

Argument Valid/Invalid

If Zeke is a cheater, then Zeke sits in the back row.	$p \Rightarrow q$
Zeke sits in the back row.	q
∴ Zeke is a cheater.	$\therefore p$

Argument Valid/Invalid

If Zeke is a cheater, then Zeke sits in the back row.	$p \Rightarrow q$
Zeke sits in the back row.	q
∴ Zeke is a cheater.	$\therefore p$
	invalid

Argument Valid/Invalid

If these two vertices are adjacent, then they do not have the same color.

These two vertices are not adjacent.

∴ These two vertices have the same color.

Argument Valid/Invalid

If these two vertices are adjacent, then they do not have the same color.

These two vertices are not adjacent.

∴ These two vertices have the same color.

$p \Rightarrow q$

p : these two vertices are adjacent

q : they do not have the same color

$\neg p$: these two vertices are not adjacent

$\neg q$: These two vertices have the same color.

$p \Rightarrow q$

$\neg p$

$\therefore \neg q$

invalid

Argument Valid/Invalid

If Canada is north of the United States, then temperatures in Canada never rise above freezing.

Canada is north of the United States..

∴ Temperatures in Canada never rise above freezing.

$p \Rightarrow q$

p

$\therefore q$

$p \Rightarrow q$

p : Canada is north of the United States

q : temperatures in Canada never rise above freezing

Argument Valid/Invalid

If New York is a big city, then New York has tall buildings $\quad p \Rightarrow q$

New York has tall buildings. $\quad q$

∴ New York is a big city. $\quad \therefore p$

p : New York is a big city

q : New York has tall buildings

Soundness of an argument

An argument is called **sound** if, and only if, it is **valid** and all its premises are **true**.

An argument that is not **sound** is called **unsound**.

Validity is a *necessary condition* for **soundness**.

Soundness is a *sufficient condition* for **validity**.

Example Argument Form

$p \lor q$

$q \Rightarrow r$

$p \land s \Rightarrow t$

$\neg r$

$\neg q \Rightarrow u \land s$

$\therefore t$

Example Argument Form

$p \vee q$	*premise*
$q \Rightarrow r$	*premise*
$p \wedge s \Rightarrow t$	*premise*
$\neg r$	*premise*
$\neg q \Rightarrow u \wedge s$	*premise*

$\therefore t$

Example Argument Form

(1)	$p \vee q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \wedge s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \wedge s$	*premise*

$\therefore t$

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*

$\therefore t$

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)

$\therefore t$

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)

$\therefore t$

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)

$\therefore t$

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)

$\therefore t$

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)
(8)	$u \land s$	Modus ponens (5,6)

$\therefore t$

Example Argument Form

(1)	$p \vee q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \wedge s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \wedge s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)
(8)	$u \wedge s$	Modus ponens (5,6)

$\therefore t$

Example Argument Form

(1)	$p \vee q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \wedge s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \wedge s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)
(8)	$u \wedge s$	Modus ponens (5,6)
(9)	s	Specialization (8)

$\therefore t$

Example Argument Form

(1)	$p \vee q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \wedge s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \wedge s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)
(8)	$u \wedge s$	Modus ponens (5,6)
(9)	s	Specialization (8)

$\therefore t$

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)
(8)	$u \land s$	Modus ponens (5,6)
(9)	s	Specialization (8)
(10)	$p \land s$	Conjunction (7,9)
	$\therefore t$	

Example Argument Form

(1)	$p \lor q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \land s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \land s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)
(8)	$u \land s$	Modus ponens (5,6)
(9)	s	Specialization (8)
(10)	$p \land s$	Conjunction (7,9)
(11)	$\therefore t$	

Example Argument Form

(1)	$p \vee q$	*premise*
(2)	$q \Rightarrow r$	*premise*
(3)	$p \wedge s \Rightarrow t$	*premise*
(4)	$\neg r$	*premise*
(5)	$\neg q \Rightarrow u \wedge s$	*premise*
(6)	$\neg q$	Modus tollen (2,4)
(7)	p	Elimination (1,6)
(8)	$u \wedge s$	Modus ponens (5,6)
(9)	s	Specialization (8)
(10)	$p \wedge s$	Conjunction (7,9)
(11)	$\therefore t$	Modus Ponens (10,3)

Topics Review

- Arguments vs Argument forms
- Laws of Inference
- Argument Validity
- Argument Soundness