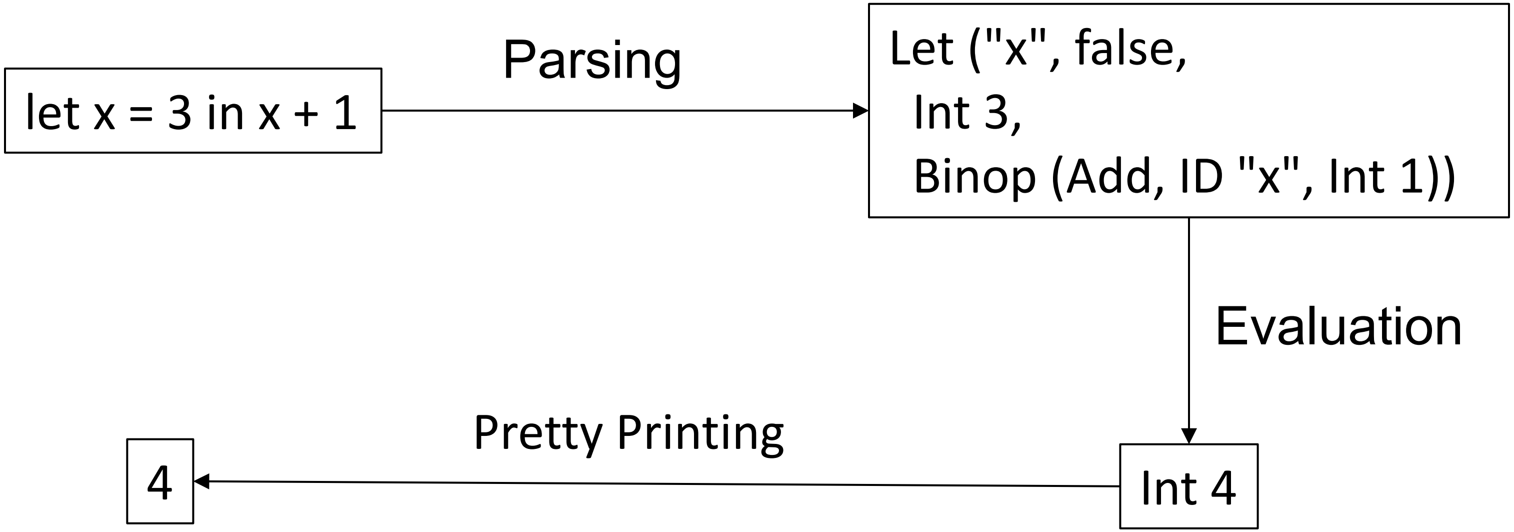
CMSC 330: Organization of Programming Languages

Type Checking

Implementing an Interpreter



let x = 3 in x + 1

Parsing

Let ("x", false,
 Int 3,
 Binop (Add, ID "x", Int 1))

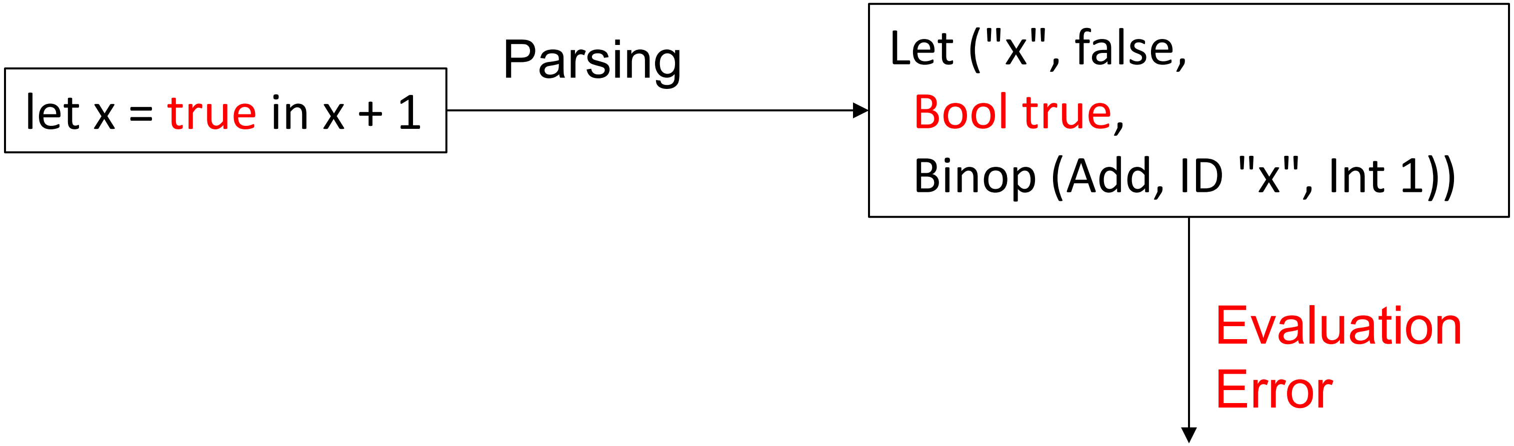
Evaluation

Int 4

Pretty Printing

4

Implementing an Interpreter: type error



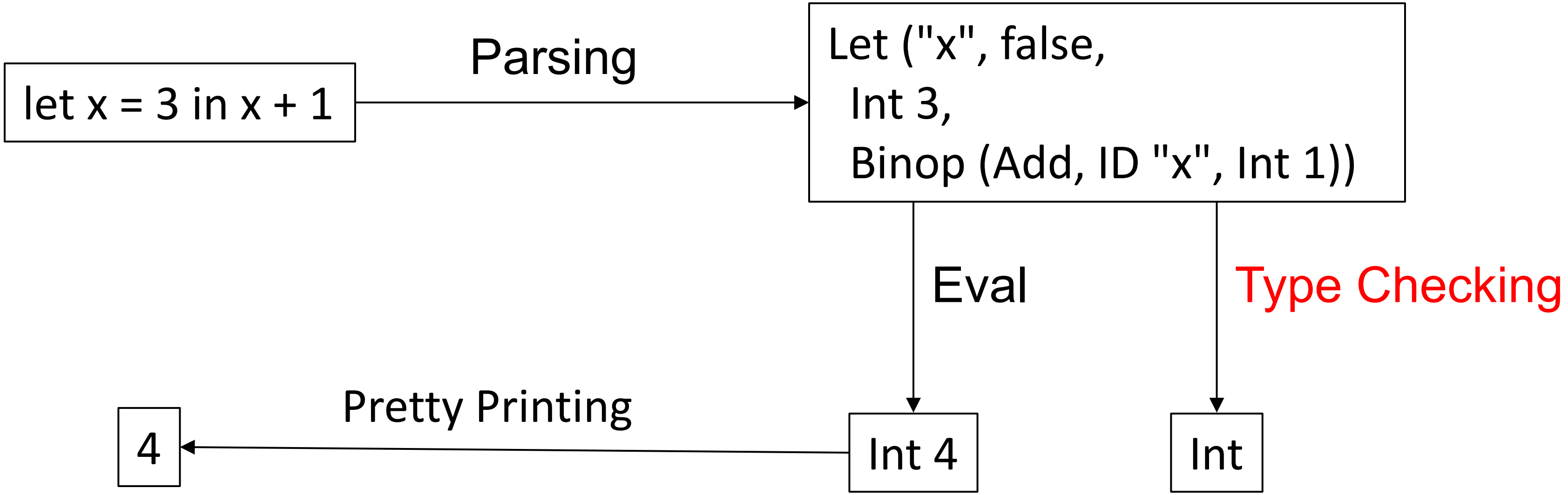
```
let x = true in x + 1
```

Parsing

```
Let ("x", false,
  Bool true,
  Binop (Add, ID "x", Int 1))
```

Evaluation Error

Type Checking



let x = 3 in x + 1

Parsing

Let ("x", false,
  Int 3,
  Binop (Add, ID "x", Int 1))

Eval

Type Checking

Pretty Printing

4

Int 4

Int

Type Systems

- A type system is a series of rules that ascribe types to expressions
  - The rules prove statements `e : t`
  - A mechanism for distinguishing good programs from bad
    - Good programs = well typed
    - Bad programs = ill-typed or not typable
    - Example:
      - 0 + 1 // well typed
      - false 0 // ill-typed: can’t apply a Boolean
      - 1 + (if true then 0 else false) // ill-typed: can’t add boolean to integer
- The process of applying these rules is called type checking
  - Or simply, typing
- Different languages have different type systems

Recall Inference Rules

- When defining how evaluation worked, we used this notation:

$$\frac{A;\ e1 \Rightarrow v1 \qquad A, x:v1;\ e2 \Rightarrow v2}{A;\ \texttt{let}\ x = e1\ \texttt{in}\ e2 \Rightarrow v2}$$

- We used inference rules to define judgment $A:e \Rightarrow v$ and translated rules into an interpreter for the MicroOCaml language.
- $A:e \Rightarrow v$ was read in English as “e evaluates to v in an Environment A

Type Checking

- Inference rules can also be used to specify a program's static semantics, I.e., the rules for type checking
- Judgment

$$G \vdash e : t$$

- is read in English as "e has type t in context G."
- We define inference rules for this judgment, just as with the operational semantics

Typing Contexts

- What is the type checking context G?
  - G is a (partial) function that maps variable names to types.

G(x) -- look up x's type in G
G,x:t -- extend G so that x maps to t

- Example context: **`x:int, y:bool, z:int`**
- When G is empty, we just write: **`e:t`**

Typing Contexts and Free Variables

- Intuition:
  - If an expression **`e`** contains free variables **`x`**, **`y`**, and **`z`** then we need to supply a context G that contains types for at least **`x`**, **`y`** and **`z`**. If we don't, we won't be able to type-check **`e`**.

```
e = Binop (Add,ID "x", Binop(Add,ID "y",ID "z"))
```

`G:`

ID	Type
`x`	`Int`
`y`	`Int`
`z`	`Int`

Type Checking Rules

- **Goal:** Give rules that define the relation "G ⊢ e : t".
  - We give one rule for every sort of expression.

```
type expr =
        Int of int
      | Bool of bool
      | ID of var
      | Fun of var * exptype * expr
      | Not of expr
      | Binop of op * expr * expr
      | If of expr * expr * expr
      | App of expr * expr
      | Let of var * bool * expr * expr
      | Record of (label * expr) list
      | Select of label * expr
```

Type Checking Rules: Booleans

- Boolean constants have type **`bool`**

$$\frac{}{G \vdash \texttt{true} : \texttt{bool}} \qquad \frac{}{G \vdash \texttt{false} : \texttt{bool}}$$

- Boolean constants b *always* have type bool, no matter what the context G is"

Type Checking Rules: Integers

- Integers have type Int

$$\frac{}{G \vdash \mathtt{n} : \mathtt{Int}}$$

- Integer constants **`n`** *always* have type Int, no matter what the context G is"

Type Checking Rules: Binary Operators

$$\frac{\mathtt{G \vdash e1:\ t1,\ G \vdash e2:\ t2,\ optype(op)=(t1,t2,t3)}}{\mathtt{G \vdash\ e1\ op\ e2:\ t3}}$$

- Where:
  - optype (+, -, *, /) = (Int, Int, Int)
  - optype (=, !=) = (‘a, ‘a , Bool)
  - optype (<, >, <=, >=) = (int, int, bool)
  - optype (&&, ||) = (Bool, Bool, Bool)

- `e1 op e2` has type `t3`, if `e1` has type `t1` ,`e2` has type `t2` and `op` is an operator that takes arguments of type `t1` and `t2` and returns a value of type `t3`

Type Checking Rules: Variables

$$\frac{}{G\vdash \mathbf{x} : G(\mathbf{x})}$$

- Variable x has the type given by the context

Type Checking Rules: Conditionals

- Eq0:

$$\frac{G \vdash e : \texttt{int}}{G \vdash \texttt{eq0}\ e : \texttt{bool}}$$

- If

$$\frac{G \vdash e1 : \texttt{bool} \qquad G \vdash e2 : t \qquad G \vdash e3 : t}{G \vdash \texttt{if}\ e1\ \texttt{then}\ e2\ \texttt{else}\ e3\ : t}$$

- If `e1` has type `bool`, and `e2` has type `t`, and `e3` has (the same) type `t` then `if e1 then e2 else e3` has type `t`

Type Checking Rules: Let

$$\frac{G \vdash e1 : t1 \qquad G, x : t1 \vdash e2 : t2}{G \vdash \texttt{let}\ x = e1\ \texttt{in}\ e2 : t2}$$

- If `e1` has type `t1` and G extended with `x:t1` proves `e2` has type `t2` then `let x = e1 in e2` has type `t2`

Type Checking Rules: Functions

$$\frac{G, x:t1 \vdash e : t2}{G \vdash \mathtt{fun}\ x:t1 \to e : t1 \to t2}$$

- if G extended with `x:t1` proves `e` has type `t2` then `fun x→e` has type `t1 → t2`

Type Checking Rules: Function Call

$$\frac{G \vdash e1 : t1 \rightarrow t2 \qquad G \vdash e2 : t1}{G \vdash e1\ e2 : t2}$$

- If `e1` has type `t1→t2` and `e2` has type `t1` then `e1 e2` has type `t2`

Type Checking Rules: Record

- Record:

$$\frac{G\vdash\ e_1:t_1\ \ldots\ G\vdash e_n:t_n}{G\ \vdash\ \{l_1=e_1\ldots l_n=e_n\}:l_1:t_1\ \ldots\ l_n:t_n}$$

- Select

$$\frac{\begin{array}{c} G\vdash\ e_1:t_1\ \ldots\ G\vdash e_n:t_n, \\ G\ \vdash\{l_1=e_1\ \ldots\ l_n=e_n\}:l_1:t_1\ \ldots\ l_n:t_n \\ G\ \vdash e:\{l_1:t_1\ \ldots\ l_n:t_n\} \end{array}}{G\ \vdash\ e.l_i:t_i}$$

Typing Derivation

- A typing derivation is a "proof" that an expression is well-typed in a particular context.
- Such proofs consist of a tree of valid rules, with no obligations left unfulfilled at the top of the tree.

$$\dfrac{\dfrac{\mathtt{G,x:int}\vdash\mathtt{x:int} \qquad \mathtt{G,x:int}\vdash\mathtt{2:int}}{\mathtt{G,x:int}\vdash\mathtt{x+2:int}}}{\mathtt{G}\vdash\mathtt{fun\ x:int}\rightarrow\mathtt{(x+2):int\text{->}int}}$$

Type Safety

- A well-typed program is accepted by the language’s type system
- A program going wrong is one that the language’s semantics gives no definition (undefined)
    - If the program were to be run, anything could happen
    - `char buf[4]; buf[4] = ‘x’; // undefined!`
- A type-safe language is one in which for every program, well-typed ⟹ well-defined
    - Or, *Well-typed programs never go wrong*, in the words of Robin Milner in 1978

Dynamic Type Checking

- The run-time checks performed by dynamic languages often called dynamic type checking
  - These languages may be said to have a dynamic type system
- The “type” of an expression checked as needed
  - Values keep tag, set when the value is created, indicating its type (e.g., what class it has)
- Disallowed operations cause run-time exception
  - Type errors may be latent in code for a long time

Quiz 1

- When is the type of a variable determined in a dynamically typed language?

- A. When the program is compiled
- B. At run-time, when the variable is used
- C. At run-time, when that variable is first assigned to
- D. At run-time, when the variable is last assigned to

Quiz 1

- When is the type of a variable determined in a dynamically typed language?

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Quiz 2

- When is the type of a variable determined in a statically typed language?

- A. When the program is compiled
- B. At run-time, when the variable is used
- C. At run-time, when that variable is first assigned to
- D. At run-time, when the variable is last assigned to

Quiz 2

- When is the type of a variable determined in a statically typed language?

- A. When the program is compiled
- B. At run-time, when the variable is used
- C. At run-time, when that variable is first assigned to
- D. At run-time, when the variable is last assigned to

Static vs. Dynamic Type Systems

- OCaml, Java, Haskell, etc. are statically typed
- Ruby, Python, etc. are dynamically typed
- But we can *view* dynamically typed languages as statically typed in a particular sense:
  - Can view all expressions as having a static type **`Dyn`**
    - The language is uni-typed
  - *All* operations are permitted on values of this type
    - E.g., in Ruby, all objects accept any method call
  - But: Some operations result in a run-time exception
    - Those not supported by the value’s dynamic “type” (tag)
    - Nevertheless, such behavior is well defined

Soundness and Completeness

- Type safety is a soundness property
  - That a term type checks implies its execution will be well-defined
- Static type systems are rarely complete
  - That a term is well-defined *does not* imply that it will type check
    - **`if true then 0 else 4+"hi"`**
- Dynamic type systems are often complete
  - *All* expressions are well defined and (statically) type check
  - **`4+"hi"`** well-defined: it gives a run-time exception

Quiz 3

- Which of the following is well-defined in OCaml, but is not well-typed?

- A. let f g = (g 1, g “hello”) in f (fun x -> x)
- B. List.map (fun x -> x + x) [1; “hello”]
- C. let x = 0 in 5 / x
- D. let x = Array.make 1 1 in x.(2)

Quiz 3

- Which of the following is well-defined in OCaml, but is not well-typed?

Functions as arguments cannot be used polymorphically

- A. let f g = (g **1**, g “**hello**”) in f (fun x -> x)
- B. List.map (fun x -> x + x) [1; “hello”]
- C. let x = 0 in 5 / x
- D. let x = Array.make 1 1 in x.(2)

Ill-typed and ill-defined

well-typed and well-defined

well-typed and well-defined

Perfect Type System? Impossible

- No type system can do all of following
  - (1) always terminate, (2) be sound, (3) be complete
  - While trying to eliminate all run-time exceptions, e.g.,
    - Using an int as a function
    - Accessing an array out of bounds
    - Dividing by zero, …
- Doing so would be undecidable
  - by reduction to the halting problem
  - Eg., **`while (…) {…} arr[-1] = 1;`**
    - *Error tantamount to proving that the while loop terminates*

Static vs. Dynamic Type Checking

Having carefully stated facts about static checking, can *now* consider arguments about which is *better*:

static checking or dynamic checking

Poll: Which Do You Prefer?

- (a) static type systems (e.g., Java, Ocaml)
- (b) dynamic type systems (e.g., Ruby, Python)

Claim 1: Dynamic is more convenient

- Dynamic typing lets you build a heterogeneous list or return a "number or a string" without workarounds

```
Ruby:    a = [1,1.5]

OCaml:
         type t =
           Int of int
         | Float of  float

         let a = [Int 1; Float 1.5];;
```

Claim 1: Static is more convenient

- Can assume data has the expected type without cluttering code with dynamic checks or having errors far from the logical mistake

Ruby:

```
def cube(x)
    if x.is_a?(Numeric)

        x * x * x
    else
        "Bad argument”
    end
end
```

OCaml:

```
let cube x = x * x * x
(* we know x is int *)
```

Claim 2: Static prevents useful programs

- Any sound static type system forbids programs that do nothing wrong

Ruby:

```
if e1 then
 “lady”
else
  [7,”hi”]
end
```

OCaml:

```
if e1 then “lady” else (7,”hi”)
(* does not type-check *)
```

Claim 2: But always workarounds

- Rather than suffer time, space, and late-errors costs of tagging everything, statically typed languages let programmers "tag as needed" (e.g., with types)

Ruby: Tags everything implicitly (uni-typed)
OCaml: Tag explicitly, as needed (code up unifying type)

```
type tort = Int of int
          | String of string
          | Cons of tort * tort
          | Fun of (tort -> tort)
          | …

if e1 then
  String "lady"
else
  Cons (Int 7, String "hi")
```

Claim 3: Static catches bugs earlier

- Static typing catches many simple bugs as soon as “compiled”.
  - Since such bugs are always caught, no need to test for them.
  - In fact, can code less carefully and “lean on” type-checker

Ruby:

```
def pow (x,y)
    if y == 0 then
         1
    else
         x * pow (y - 1)

    end
end
# can’t detect until run
```

OCaml:

```
let pow x y =
if y = 0  then 1
else x * pow (y-1)

(* does not type-check *)
```

Claim 3: Static catches only easy bugs

- But static often catches only “easy” bugs, so you still have to test your functions, which should find the “easy” bugs too

Ruby:

```
def pow (x,y)
    if y == 0 then
         1
    else
         x + pow (x,(y-1))
    end
end
```

OCaml:

```
let pow x y =
if y = 0  then 1
else x + pow x (y-1)

(* oops *)
```

Claim 4: Static typing is faster

- Language implementation:
  - Does not need to store tags (space, time)
  - Does not need to check tags (time)
  - Can rely on values being a particular type, so it can perform more optimizations
- Your code:
  - Does not need to check arguments and results beyond what is evidently required

Claim 4: Dynamic typing is not too much slower

- Language implementation:
  - Can use remove some unnecessary tags and tests despite the lack of types
    - While difficult (impossible) in general, it is often possible for the performance-critical parts of a program
- Your code:
  - Do not need to “code around” type-system limitations with extra tags, functions etc.

Claim 5: Code reuse easier with dynamic

Without a restrictive type system, more code can just be reused with data of different types

- If you use cons cells for everything, libraries that work on cons cells are useful
- Collections libraries are amazingly useful but often have very complicated static types
  - Polymorphism/generics/etc. are hard to understand, but are aiming to provide what dynamic typing gives naturally
- Etc.

Claim 5: Code reuse easier with static

The type system serves as “checked documentation,” making the “contract” with others’ code easier to understand and use correctly

Redux: Which Do You Prefer?

- (a) static type systems (e.g., Java, Ocaml)
- (b) dynamic type systems (e.g., Ruby, Python)

Static vs. Dynamic: Age-old Debate

- Static vs. dynamic typing is too coarse a question
  - Better question: *What* should we enforce statically?
    - E.g., OCaml checks array bounds, division-by-zero, at run-time
  - Legitimate trade-offs

- Idea: Flexible languages allowing best-of-both-worlds?
  - Use static types in some parts of the program, but dynamic checking in other parts?
    - Called gradual typing: an idea still under active research
  - Would programmers use such flexibility well? Who decides?