Discrete Structures

Conditionals cont.
Laws of Equivalence

Review

- Truth Tables
  - # rows
  - Order of rows
- Propositional Logic
  - Precedence
  - Two parts of a conditional
  - 2 statements logically equivalent?
  - Tautology
  - Contradiction

Review

Conditional Statement

p	q	$p \Rightarrow q$
0	0	
0	1	
1	0	
1	1	

Review

Conditional Statement

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Review

If today is Easter, then tomorrow is Monday.

$p \Rightarrow q$:

Converse:

Inverse:

Contrapositive:

Review

If today is Easter, then tomorrow is Monday.

$p \Rightarrow q$: Today is Easter implies tomorrow is Monday.

Converse: If tomorrow is Monday, then today is Easter.

Inverse: If today is not Easter, then tomorrow is not Monday.

Contrapositive: If tomorrow is not Monday, then today is not Easter.

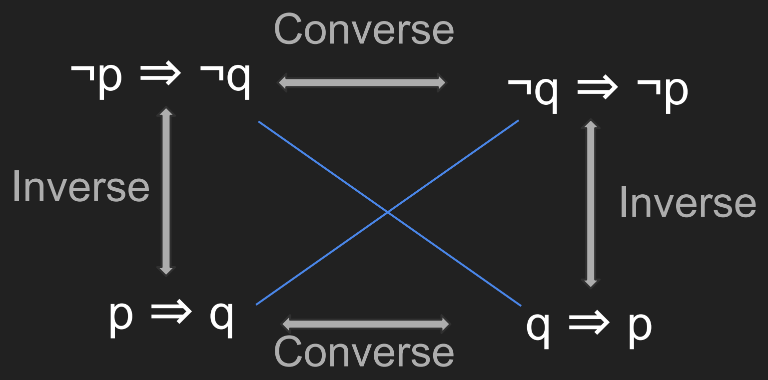
Relationship between inverse converse?

$p \Rightarrow q$

Converse: $q \Rightarrow p$

Inverse: $\neg p \Rightarrow \neg q$

Relationship between inverse converse



Review

Is this true or false?

If 0 = 1 then 1 = 2

Are these statements valid (true) or invalid (false)

A. If computers are machines, then Babe Ruth was a baseball player.
B. If $2 + 2 = 5$, then Babe Ruth was a baseball player.
C. If $2 + 2 = 5$, then Mickey Mouse is president.

Conditional Statement Equivalence

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Conditional Statement Equivalence

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Conditional Statement Equivalence

p	q	$p \Rightarrow q$	$\neg(p \land \neg q)$
0	0	1	
0	1	1	
1	0	0	
1	1	1	

Conditional Statement Equivalence

p	q	$\neg q$	$(p \land \neg q)$	$p \Rightarrow q$	$\neg(p \land \neg q)$
0	0	1		1	
0	1	0		1	
1	0	1		0	
1	1	0		1	

Conditional Statement Equivalence

p	q	$\neg q$	$(p \land \neg q)$	$p \Rightarrow q$	$\neg(p \land \neg q)$
0	0	1	0	1	
0	1	0	0	1	
1	0	1	1	0	
1	1	0	0	1	

Conditional Statement Equivalence

p	q	$\neg q$	$(p \land \neg q)$	$p \Rightarrow q$	$\neg(p \land \neg q)$
0	0	1	0	1	1
0	1	0	0	1	1
1	0	1	1	0	0
1	1	0	0	1	1

Conditional Statement Equivalence

p	q	$\neg q$	$(p \land \neg q)$	$p \Rightarrow q$	$\neg(p \land \neg q)$
0	0	1	0	1	1
0	1	0	0	1	1
1	0	1	1	0	0
1	1	0	0	1	1

Conditional Statement Equivalence

p	q	$p \Rightarrow q$	$\neg p \vee q$
0	0	1	
0	1	1	
1	0	0	
1	1	1	

Conditional Statement Equivalence

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
0	0	1	1	
0	1	1	1	
1	0	0	0	
1	1	1	0	

Conditional Statement Equivalence

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

Conditional Statement Equivalence

$p \Rightarrow q \equiv (\neg p \vee q)$

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

Conditional Statement Equivalence

$p \Rightarrow q \equiv (\neg p \vee q)$

If you do not get to work on time, then you are fired.

p : You do NOT get to work on time

q : You are fired.

¬p: You get to work on time.

p	q	$p \Rightarrow q$	$\neg p$	$(\neg p \vee q)$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

Review

Conditional Statement

$p \Rightarrow q \equiv \neg(p \land \neg q) \equiv (\neg p \lor q)$

p	q	$p \Rightarrow q$	$\neg(p \land \neg q)$	$(\neg p \lor q)$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	1	1

Conditionals

What is the negation of an implication?

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Conditionals

What is the negation of an implication?

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$
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0	1	1	
1	0	0	
1	1	1	

Conditionals

What is the negation of an implication?

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

Conditionals

What is the negation of an implication?

$p \land \neg q \equiv \neg(p \Rightarrow q)$

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

Conditionals

What is the negation of an implication?

$p \wedge \neg q \equiv \neg(p \Rightarrow q)$

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

Biconditional

p	q	$p \Leftrightarrow q$
0	0	
0	1	
1	0	
1	1	

Biconditional

p	q	$p \Leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Biconditional

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Rightarrow q \ \wedge \ q \Rightarrow p$
0	0			
0	1			
1	0			
1	1			

Biconditional

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Rightarrow q \land q \Rightarrow p$
0	0	1		
0	1	1		
1	0			
1	1			

Biconditional

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Rightarrow q \land q \Rightarrow p$
0	0	1		
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Biconditional

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Rightarrow q \ \land \ q \Rightarrow p$
0	0	1	1	1
0	1	1	0	
1	0	0	1	
1	1	1	1	1

Biconditional

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Rightarrow q \ \wedge \ q \Rightarrow p$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

Biconditional

This computer program is correct if, and only if, it produces correct answers for all possible sets of input data.

$p \Rightarrow q$

$q \Rightarrow p$

Conditionals

Necessary

Sufficient

Conditionals

Sufficient

r is a **sufficient condition** for s means “if r then s.”

Conditionals

Necessary

r is a **necessary condition** for s means "if not r then not s."

$\neg r \Rightarrow \neg s$

What is its contrapositive?

Recall the **contrapositive** of $p \Rightarrow q$ is: $\boldsymbol{\neg q \Rightarrow \neg p}$

$\neg(\neg s) \Rightarrow \neg(\neg r)$

$s \Rightarrow r$

Conditionals

Sufficient

r is a **sufficient condition** for s means "if r then s."

$r \Rightarrow s$

Necessary

r is a **necessary condition** for s means "if not r then not s."

$\neg r \Rightarrow \neg s$

$s \Rightarrow r$

What does it mean if r is **necessary** and **sufficient** for s?

What is necessary? and sufficient?

If John is eligible to vote, then he is at least 18 years old.

John is eligible to vote if and only if he is at least 18 years old.

~~John is eligible to vote if and only if he is at least 18 years old~~.

Convert this into two conditional statements

If you are age 35 or older is a necessary condition for you being president of the United States.

Convert this into two conditional statements

If you are age 35 or older is a necessary condition for you being president of the United States.

p : You are age 35 or older

q : You are president of the United States

p is a necessary condition for q.

$\neg p \Rightarrow \neg q$

- If you are NOT 35 or older then you are NOT president of the United States.
- If you are president of the United States, then you are older than 35.

How can we prove logical equivalence?

- Same truth tables
- Algebraically (applying laws of equivalence)

Laws of Equivalence

1. Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity	$p \wedge 1 \equiv p$	$p \vee 0 \equiv p$
5. Negation	$p \vee \neg p \equiv 1$	$p \wedge \neg p \equiv 0$
6. Double negative	$\neg(\neg p) \equiv p$	
7. Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
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9. De Morgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
10. Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
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De Morgan’s Law

$\neg(p \land q) \equiv \neg p \lor \neg q$

p	q	$\neg(p \land q)$	$\neg p \lor \neg q$
0	0		
0	1		
1	0		
1	1		

De Morgan’s Law

$\neg(p \land q) \equiv \neg p \lor \neg q$

p	q	(p ∧ q)	¬p	¬q	¬(p ∧ q)	¬p ∨ ¬q
0	0	0	1	1	1	1
0	1	0	1	0	1	1
1	0	0	0	1	1	1
1	1	1	0	0	0	0

Review

Conditional Statement

$p \Rightarrow q \equiv \neg(p \wedge \neg q) \equiv (\neg p \vee q)$

p	q	$p \Rightarrow q$	$\neg(p \wedge \neg q)$	$(\neg p \vee q)$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	1	1

Laws of Equivalence

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Simplifying Statements

Style Guide:

Proving logical equivalence (eg. prove $a \equiv b$)

$$
\begin{array}{lll}
 & \text{starting statement} & \\
\equiv & \text{derived statement} & \text{justification} \\
 & \vdots & \\
\hline
\equiv & \text{ending statement} & \text{justification}
\end{array}
$$

Style Guide so far

- False = 0, True = 1
- Order your truth table rows: 00,01,10,11
- Proving logical equivalence (eg. prove $a \equiv b$)

	starting statement	
$\equiv$	derived statement	justification
	$\vdots$	
$\equiv$	ending statement	justification

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

$$\neg(\neg p \land q) \land (p \lor q)$$

$\equiv$

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

$$\neg(\neg p \land q) \land (p \lor q)$$

$\equiv$ De Morgan's Law

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

$$\neg(\neg p \land q) \land (p \lor q)$$

$$\equiv \quad (\neg(\neg p) \lor \neg q) \land (p \lor q) \qquad \text{De Morgan's Law}$$

Prove: $\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$

	$\neg(\neg p \wedge q) \wedge (p \vee q)$	
$\equiv$	$(\neg(\neg p) \vee \neg q) \wedge (p \vee q)$	De Morgan's Law
$\equiv$		

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan’s Law
$\equiv$		Double Negative Law

Prove: $\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$

$$\neg(\neg p \wedge q) \wedge (p \vee q)$$

$$\equiv (\neg(\neg p) \vee \neg q) \wedge (p \vee q)$$ De Morgan's Law

$$\equiv (p \vee \neg q) \wedge (p \vee q)$$ Double Negative Law

$$\equiv$$

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan's Law
$\equiv$	$(p \lor \neg q) \land (p \lor q)$	Double Negative Law
$\equiv$		Distributive Law

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan's Law
$\equiv$	$(p \lor \neg q) \land (p \lor q)$	Double Negative Law
$\equiv$	$p \lor (\neg q \land q)$	Distributive Law
$\equiv$		

Prove: $\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$

	$\neg(\neg p \wedge q) \wedge (p \vee q)$	
$\equiv$	$(\neg(\neg p) \vee \neg q) \wedge (p \vee q)$	De Morgan's Law
$\equiv$	$(p \vee \neg q) \wedge (p \vee q)$	Double Negative Law
$\equiv$	$p \vee (\neg q \wedge q)$	Distributive Law
$\equiv$		Commutative Law

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan's Law
$\equiv$	$(p \lor \neg q) \land (p \lor q)$	Double Negative Law
$\equiv$	$p \lor (\neg q \land q)$	Distributive Law
$\equiv$	$p \lor (q \land \neg q)$	Commutative Law

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan's Law
$\equiv$	$(p \lor \neg q) \land (p \lor q)$	Double Negative Law
$\equiv$	$p \lor (\neg q \land q)$	Distributive Law
$\equiv$	$p \lor (q \land \neg q)$	Commutative Law
$\equiv$		Negation Law

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan’s Law
$\equiv$	$(p \lor \neg q) \land (p \lor q)$	Double Negative Law
$\equiv$	$p \lor (\neg q \land q)$	Distributive Law
$\equiv$	$p \lor (q \land \neg q)$	Commutative Law
$\equiv$	$p \lor 0$	Negation Law

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan's Law
$\equiv$	$(p \lor \neg q) \land (p \lor q)$	Double Negative Law
$\equiv$	$p \lor (\neg q \land q)$	Distributive Law
$\equiv$	$p \lor (q \land \neg q)$	Commutative Law
$\equiv$	$p \lor 0$	Negation Law
$\equiv$		Identity Law

Prove: $\neg(\neg p \land q) \land (p \lor q) \equiv p$

	$\neg(\neg p \land q) \land (p \lor q)$	
$\equiv$	$(\neg(\neg p) \lor \neg q) \land (p \lor q)$	De Morgan's Law
$\equiv$	$(p \lor \neg q) \land (p \lor q)$	Double Negative Law
$\equiv$	$p \lor (\neg q \land q)$	Distributive Law
$\equiv$	$p \lor (q \land \neg q)$	Commutative Law
$\equiv$	$p \lor 0$	Negation Law
$\equiv$	p	Identity Law

Laws of Equivalence

1. Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity	$p \wedge 1 \equiv p$	$p \vee 0 \equiv p$
5. Negation	$p \vee \neg p \equiv 1$	$p \wedge \neg p \equiv 0$
6. Double negative	$\neg(\neg p) \equiv p$	
7. Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound	$p \vee 1 \equiv 1$	$p \wedge 0 \equiv 0$
9. De Morgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
10. Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of t and c	$\neg 1 \equiv 0$	$\neg 0 \equiv 1$

De Morgan’s Law

$$\neg(p \land q \lor r)$$

$$\equiv$$

Incorrect

$$\neg(p \land q \lor r)$$

$$\equiv \quad \neg p \lor \neg q \land \neg r$$

Post lecture correction from:

$p \lor \neg q \land \neg r$ to

$\neg p \lor \neg q \land \neg r$

Correct

$$\neg(p \wedge q \vee r)$$

$$\equiv \quad (\neg p \vee \neg q) \wedge \neg r$$

Post lecture correction from:

$(p \vee \neg q) \wedge \neg r$ to

$(\neg p \vee \neg q) \wedge \neg r$

Post Lecture Addition

$$
\begin{aligned}
& \neg(p \land q \lor r) & \\
\equiv\ & \neg((p \land q) \lor r) & \text{Precedence Clarification} \\
\equiv\ & (\neg(p \lor q) \land \neg r) & \text{De Morgan's Law} \\
\equiv\ & (\neg p \lor \neg q) \land \neg r & \text{De Morgan's Law (final parens optional)}
\end{aligned}
$$

De Morgan’s Law

Apply De Morgan’s Law to:

$p \wedge q$

De Morgan's Law

Apply De Morgan's Law to:

$p \land q \equiv \neg(\neg p \lor \neg q)$

De Morgan’s Law

Apply De Morgan’s Law to:

$p \land \neg q \lor r \equiv$

De Morgan’s Law

Apply De Morgan’s Law to:

$p \wedge \neg q \vee r \equiv \neg((\neg p \vee q) \wedge \neg r)$

De Morgan's Law

Apply De Morgan's Law to:

$p \wedge \neg q \vee r \equiv \neg((\neg p \vee q) \wedge \neg r)$

$p \wedge \neg q \vee r$

$\equiv$

Precedence Clarification

De Morgan's Law

Apply De Morgan's Law to:

$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

$p \land \neg q \lor r$

$\equiv (p \land \neg q) \lor r$ Precedence Clarification

De Morgan's Law

Apply De Morgan's Law to:

$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$		Double Negative Law

De Morgan's Law

Apply De Morgan's Law to:

$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law

De Morgan's Law

Apply De Morgan's Law to:

$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$		De Morgan's Law

De Morgan’s Law

Apply De Morgan’s Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan’s Law
$\equiv$		

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$		De Morgan's Law
$\equiv$		

De Morgan's Law

Apply De Morgan's Law to:

$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan's Law
$\equiv$		

De Morgan's Law

Apply De Morgan's Law to:
$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan's Law
$\equiv$		

De Morgan's Law

Apply De Morgan's Law to:

$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan's Law
$\equiv$		Double Negative Law

De Morgan's Law

Apply De Morgan's Law to:

$p \land \neg q \lor r \equiv \neg((\neg p \lor q) \land \neg r)$

	$p \land \neg q \lor r$	
$\equiv$	$(p \land \neg q) \lor r$	Precedence Clarification
$\equiv$	$\neg\neg((p \land \neg q) \lor r)$	Double Negative Law
$\equiv$	$\neg(\neg(p \land \neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor \neg\neg q) \land \neg r)$	De Morgan's Law
$\equiv$	$\neg((\neg p \lor q) \land \neg r)$	Double Negative Law

Practice

Prove

$(p \lor \neg q) \land (\neg p \lor \neg q) \equiv \neg q$

	$(p \lor \neg q) \land (\neg p \lor \neg q)$	
$\equiv$	$(\neg q \lor p) \land (\neg q \lor \neg p)$	Commutative Law
	$\neg q \lor (p \land \neg p)$	Distributive Law
	$\neg q \lor 0$	Negation Law
	$\neg q$	Identity Law