Discrete Structures

Precedence, Truth Tables, Implications
9/1/2022

Review

- Statements
- Variables

$x > 30$

Let x be > 20 and < 50

x > 30

Today

- Precedence
- Truth Tables
- Conditionals/Implications

Precedence

3 operators: ¬, ∧, ∨

$\neg p \lor q \land r$

$(\neg p) \lor (q \land r)$

Precedence

Highest/strongest to lowest/weakest
¬
∧
∨
⇒ ⇔

Precedence

3 operators: $\neg$, $\land$, $\lor$

$\neg p \lor q \land r$

$(\neg p) \lor (q \land r)$

Precedence **Analogy**

Highest/strongest

$\neg$	^ (exp)
$\land$	* (mul)
$\lor$	+ (add)
$\Rightarrow$ $\Leftrightarrow$	

lowest/weakest

Practice makes Precedence

Convention

True = 1

False = 0

Convention

True = 1

False = 0

Why don’t we use ‘t’ and ‘f’ for truth values?

Truth tables

p	¬p

Truth tables

p	$\neg p$
0	
1	

Truth tables

p	¬p
0	1
1	

Truth tables

p	¬p
0	1
1	0

Truth tables $p \wedge q$

What are all the possible combinations of the values p and q could be?

p	q	$p \wedge q$

Truth tables $p \wedge q$

What are all the possible combinations of the values p and q could be?

Both true

Both false

First true, second false

first false, second true

p	q	$p \wedge q$

Truth tables $p \wedge q$

What are all the possible combinations of the values p and q could be?

Both true

Both false

First true, second false

first false, second true

p	q	$p \wedge q$
0	0	
0	1	
1	0	
1	1	

Truth tables $p \land q$

What are all the possible combinations of the values p and q could be?

Both true

Both false

First true, second false

first false, second true

p	q	$p \land q$
0	0	0
0	1	0
1	0	0
1	1	1

Truth tables $p \lor q$

p	q	$p \lor q$
0	0	
0	1	
1	0	
1	1	

Truth tables $p \vee q$

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Consistency check:

The order of truth table rows.

p	q	y
0	0	
0	1	
1	0	
1	1	

Consistency check:

The order of truth table rows.

- Always go “smallest” to “largest”
- `00,01,10,11`
- `00 < 01 < 10 < 11`

p	q	y
0	0	
0	1	
1	0	
1	1	

Truth table number of rows

1 variable, 2 rows

2 variables, 4 rows

3 variables, _ rows

Truth table number of rows

p	y
0	
1	

p	q	y
0	0	
0	1	
1	0	
1	1	

Truth table number of rows

p	y
0	
1	

p	q	*y*
0	0	
0	1	
1	0	
1	1	

Truth table number of rows

1 variable

p	y
0	
1	

2 variables

p	q	y
0	0	
0	1	
1	0	
1	1	

3 variables

p	q	r	y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Truth table number of rows

p	y
0	
1	

p	q	y
0	0	
0	1	
1	0	
1	1	

p	q	r	y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Truth table number of rows

p	y
0	
1	

p	q	y
0	0	
0	1	
1	0	
1	1	

p	q	r	y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Truth table number of rows

p	y
0	
1	

p	q	y
0	0	
0	1	
1	0	
1	1	

p	q	r	y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Truth table number of rows

p	y
0	
1	

p	q	y
0	0	
0	1	
1	0	
1	1	

p	q	r	y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Truth table number of rows

p	y
0	
1	

p	q	y
0	0	
0	1	
1	0	
1	1	

p	q	r	y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

How many rows will be in this truth table?

p	q	r	s	y

16

$2^4 = 16$

Finish the truth table for: $p \wedge \neg p$

Finish the truth table for: $p \land \neg p$

p	$p \land \neg p$
0	
1	

Finish the truth table for: $p \land \neg p$

p	$\neg p$	$p \land \neg p$
0	1	0
1	0	0

Finish the truth table for: $p \land \neg p$

Contradiction

$p \land \neg p \equiv 0$

p	$\neg p$	$p \land \neg p$
0	1	0
1	0	0

Contradiction

A **contradiction** is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables

p	$\neg p$	$p \land \neg p$
0	1	0
1	0	0

Equivalence

Two statements are called **logically equivalent** if, and only if, they have identical truth values for each possible substitution of statements for their statement variables.

Written: $p \equiv q$

Finish the truth table for: $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
0	1	
1	0	

Finish the truth table for: $p \lor \neg p$

$p \lor \neg p \equiv 1$

p	$\neg p$	$p \lor \neg p$
0	1	1
1	0	1

Finish the truth table for: $p \vee \neg p$

Tautology

p	$\neg p$	$p \vee \neg p$
0	1	1
1	0	1

Tautology

A **tautology** is a statement that is always true regardless of the truth values of the individual statements substituted for its statement variables.

p	$\neg p$	$p \vee \neg p$
0	1	1
1	0	1

Exclusive OR

Truth table

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive OR

Truth table

p	q	$p \oplus q$
0	0	
0	1	
1	0	
1	1	

Equivalence

Two statements are called **logically equivalent** if, and only if, they have identical truth values for each possible substitution of statements for their statement variables.

Written: $p \equiv q$

Exclusive OR

$$\overbrace{p \oplus q} \equiv ? \overbrace{(p \wedge \neg q) \vee (\neg p \wedge q)} \equiv ? \overbrace{(p \vee q) \wedge \neg(p \wedge q)}$$

They are equivalent iff they have the same truth table column values

Proving Equivalence

p	q	$(p \land \neg q) \lor (\neg p \land q)$	$(p \lor q) \land \neg(p \land q)$
0	0		
0	1		
1	0		
1	1		

Proving Equivalence

p	q	$(p \land \neg q) \lor (\neg p \land q)$
0	0	
0	1	
1	0	
1	1	

Proving Equivalence

p	q	$(p \land \neg q)$	$(\neg p \land q)$	$(p \land \neg q) \lor (\neg p \land q)$
0	0			
0	1			
1	0			
1	1			

Proving Equivalence

p	q	$\neg p$	$\neg q$	$(p \land \neg q)$	$(\neg p \land q)$	$(p \land \neg q) \lor (\neg p \land q)$
0	0					
0	1					
1	0					
1	1					

Proving Equivalence

p	q	$\neg p$	$\neg q$	$(p \land \neg q)$	$(\neg p \land q)$	$(p \land \neg q) \lor (\neg p \land q)$
0	0					
0	1					
1	0					
1	1					

Proving Equivalence

p	q	$\neg p$	$\neg q$	$(p \land \neg q)$	$(\neg p \land q)$	$(p \land \neg q) \lor (\neg p \land q)$
0	0	1				
0	1	1				
1	0	0				
1	1	0				

Proving Equivalence

p	q	¬p	¬q	(p ∧ ¬q)	(¬p ∧ q)	(p ∧ ¬q) ∨ (¬p ∧ q)
0	0	1				
0	1	1				
1	0	0				
1	1	0				

Proving Equivalence

p	q	¬p	¬q	(p ∧ ¬q)	(¬p ∧ q)	(p ∧ ¬q) ∨ (¬p ∧ q)
0	0	1	1			
0	1	1	0			
1	0	0	1			
1	1	0	0			

Proving Equivalence

p	q	¬p	¬q	(p ∧ ¬q)	(¬p ∧ q)	(p ∧ ¬q) ∨ (¬p ∧ q)
0	0	1	1			
0	1	1	0			
1	0	0	1			
1	1	0	0			

Proving Equivalence

p	q	¬p	¬q	(p ∧ ¬q)	(¬p ∧ q)	(p ∧ ¬q) ∨ (¬p ∧ q)
0	0	1	1	0		
0	1	1	0	0		
1	0	0	1	1		
1	1	0	0	0		

Proving Equivalence

p	q	$\neg p$	$\neg q$	$(p \land \neg q)$	$(\neg p \land q)$	$(p \land \neg q) \lor (\neg p \land q)$
0	0	1	1	0		
0	1	1	0	0		
1	0	0	1	1		
1	1	0	0	0		

Proving Equivalence

p	q	¬p	¬q	(p ∧ ¬q)	(¬p ∧ q)	(p ∧ ¬q) ∨ (¬p ∧ q)
0	0	1	1	0	0	
0	1	1	0	0	1	
1	0	0	1	1	0	
1	1	0	0	0	0	

Proving Equivalence

p	q	¬p	¬q	(p ∧ ¬q)	(¬p ∧ q)	(p ∧ ¬q) ∨ (¬p ∧ q)
0	0	1	1	0	0	
0	1	1	0	0	1	
1	0	0	1	1	0	
1	1	0	0	0	0	

Proving Equivalence

p	q	¬p	¬q	(p ∧ ¬q)	(¬p ∧ q)	(p ∧ ¬q) ∨ (¬p ∧ q)
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

Proving Equivalence

p	q	$(p \wedge \neg q) \vee (\neg p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
0	0	0	
0	1	1	
1	0	1	
1	1	0	

Proving Equivalence

p	q	$(p \lor q) \land \neg(p \land q)$
0	0	
0	1	
1	0	
1	1	

Proving Equivalence

p	q	$(p \wedge q)$	$(p \vee q)$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
0	0				
0	1				
1	0				
1	1				

Proving Equivalence

p	q	$(p \land q)$	$(p \lor q)$	$\neg(p \land q)$	$(p \lor q) \land \neg(p \land q)$
0	0				
0	1				
1	0				
1	1				

Proving Equivalence

p	q	$(p \land q)$	$(p \lor q)$	$\neg(p \land q)$	$(p \lor q) \land \neg(p \land q)$
0	0	0			
0	1	0			
1	0	0			
1	1	1			

Proving Equivalence

p	q	(p ∧ q)	(p ∨ q)	¬(p ∧ q)	(p ∨ q) ∧ ¬(p ∧ q)
0	0	0			
0	1	0			
1	0	0			
1	1	1			

Proving Equivalence

p	q	(p ∧ q)	(p ∨ q)	¬(p ∧ q)	(p ∨ q) ∧ ¬(p ∧ q)
0	0	0	0		
0	1	0	1		
1	0	0	1		
1	1	1	1		

Proving Equivalence

p	q	$(p \land q)$	$(p \lor q)$	$\neg(p \land q)$	$(p \lor q) \land \neg(p \land q)$
0	0	0	0		
0	1	0	1		
1	0	0	1		
1	1	1	1		

Proving Equivalence

p	q	$(p \wedge q)$	$(p \vee q)$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
0	0	0	0	1	
0	1	0	1	1	
1	0	0	1	1	
1	1	1	1	0	

Proving Equivalence

p	q	$(p \wedge q)$	$(p \vee q)$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
0	0	0	0	1	
0	1	0	1	1	
1	0	0	1	1	
1	1	1	1	0	

Proving Equivalence

p	q	$(p \land q)$	$(p \lor q)$	$\neg(p \land q)$	$(p \lor q) \land \neg(p \land q)$
0	0	0	0	1	0
0	1	0	1	1	1
1	0	0	1	1	1
1	1	1	1	0	0

Types of statements so far

- Negations
- Conjunctions
- Disjunctions

- **Conditionals/implications**

Conditional Statements/implications

English:

- "If p then q"
- "q, if p"

Definition: p *implies* q

Written: $p \Rightarrow q$

Conditional Statements/implications

English:

- "If p then q"
- "q, if p"

Definition: p *implies* q

Written: $p \Rightarrow q$

If it is raining ***then*** it is cloudy.

It is cloudy ***if*** it is raining.

It is raining ***implies*** it is cloudy.

p : it is raining

q : it is cloudy

$s : p \Rightarrow q$

Conditional Statements/implications

English:

- "If p then q"
- "q, if p"

Definition: p *implies* q

Written: $p \Rightarrow q$

If I am in 250 lecture ***then*** it is Tuesday or Thursday.

It is Tuesday or Thursday ***if*** I am in 250 lecture.

I am in 250 lecture ***implies*** it is Tuesday or Thursday.

p : I am in 250 lecture

q : it is Tuesday or Thursday.

$s : p \Rightarrow q$

Conditional Statements/implications

Put another way…

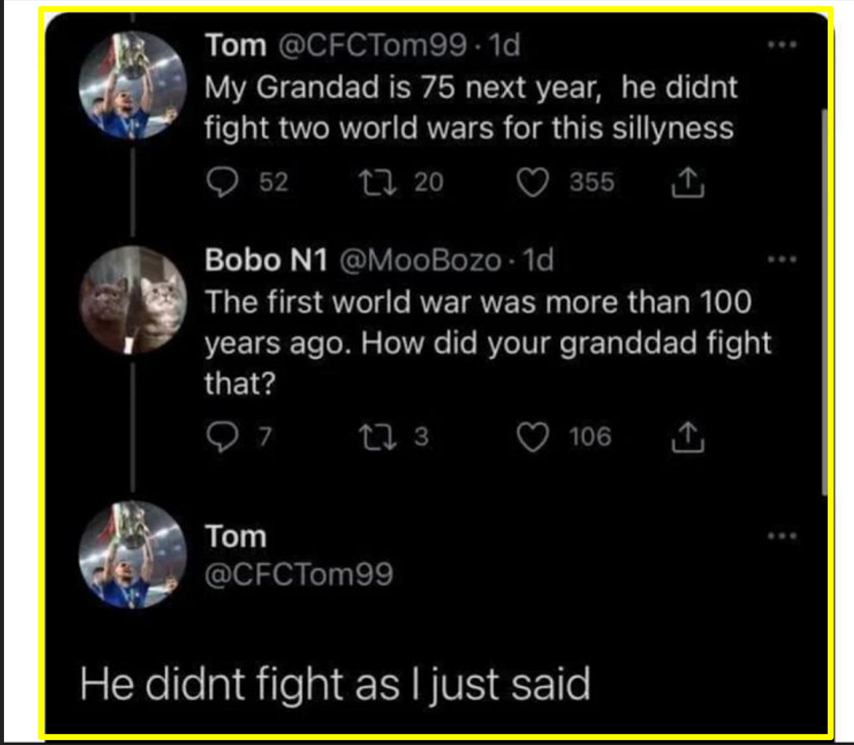
IF my grandad fought 2 world wars ***then*** he didn't fight for this silliness.

p : my grandad fought 2 world wars

q : He didn't fight for this silliness

$s : p \Rightarrow q$

$r : \neg p$



If 0, then: anything you want!

$p \Rightarrow q$

if p is false (0), $p \Rightarrow q$ is vacuously true

Every time I go to mars, I bring 12 pink elephants.

If I go to Mars, **then** I bring 12 pink elephants.

I go to Mars = 0.

Vacuously true.

Truth table for: $\mathrm{p} \Rightarrow \mathrm{q}$

p : it is raining

q : it is cloudy

p	q	$p \Rightarrow q$
0	0	
0	1	
1	0	
1	1	

Truth table for: $p \Rightarrow q$

p : it is raining

q : it is cloudy

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Converse

If $\mathbf{p} \Rightarrow \mathbf{q}$,

Does it follow that $\mathbf{q} \Rightarrow \mathbf{p}$?

Eg.

p : I am in 250 lecture.

q : It is Tuesday or Thursday.

No!

Converse

$q \Rightarrow p$ is called the **converse** of $p \Rightarrow q$

Inverse

$p \Rightarrow q$

Does it follow that

$\neg p \Rightarrow \neg q$?

p : I am in 250 lecture.

q : It is Tuesday or Thursday.

$\neg p$: I am NOT in 250 lecture.

$\neg q$: It is NOT Tuesday or Thursday.

$\neg p \Rightarrow \neg q$ is the **inverse** of $p \Rightarrow q$

Contrapositive

$p \Rightarrow q$

Does it follow that

$\neg q \Rightarrow \neg p$?

p : I am in 250 lecture.

q : It is Tuesday or Thursday.

$\neg p$: I am NOT in 250 lecture.

$\neg q$: It is NOT Tuesday or Thursday.

$\neg q \Rightarrow \neg p$ is the **contrapositive** of $p \Rightarrow q$

Converse, Inverse, Contrapositive

	Converse	*Inverse*	*Contrapositive*
$p \Rightarrow q$	$q \Rightarrow p$	$\neg p \Rightarrow \neg q$	$\neg q \Rightarrow \neg p$
If $p \Rightarrow q$*, does it hold?*	NO	NO	YES

$$(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$$

Topic review

- Precedence
- Truth tables
- Tautology
- Contradiction
- Equivalence
- Implication/Conditional
    - Inverse, Converse, Contrapositive