

# CMSC330 - Organization of Programming Languages Spring 2025- FinalSolutions

CMSC330 Course Staff  
University of Maryland  
Department of Computer Science

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*I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination*

Signature: \_\_\_\_\_

## Ground Rules

- Please write legibly. **If we cannot read your answer you will not receive credit.**
- You may use anything on the accompanying reference sheet anywhere on this exam
- Please remove the reference sheet from the exam
- You may not leave the room or hand in your exam within the last 10 minutes of the exam
- If anything is unclear, ask a proctor. If you are still confused, write down your assumptions in the margin
- Do not take photos of this exam or share this exam in any way shape or form
- If you need extra space, the last page is for scratch work. Make a note for the grader to check the scratch page if you want it graded.
- NOTE: there is a page for scratch work at the end of the exam. You may use this freely, just don't tear it off.

Question	Points
P1.	10
P2.	5
P3.	6
P4.	4
P5.	6
P6.	4
P7.	10
P8.	10
P9.	8
P10.	6
P11.	8
P12.	8
P13.	10
P14.	5
EC	2
Total	100 + 2

## Problem 1: Concepts

[Total 10 pts]

	true	false
Every CFG can be represented by an NFA	<input type="radio"/>	<input checked="" type="radio"/>
If $A <: B$ and $B <: C$ then $A <: C$	<input checked="" type="radio"/>	<input type="radio"/>
In Rust, every variable is immutable by default	<input checked="" type="radio"/>	<input type="radio"/>
OCaml is dynamically typed because it uses type inference	<input type="radio"/>	<input checked="" type="radio"/>
Regular expressions are necessary to code a parser.	<input type="radio"/>	<input checked="" type="radio"/>
Safe Rust will never produce runtime errors.	<input type="radio"/>	<input checked="" type="radio"/>
By fixing all logic bugs, you have a secure program.	<input type="radio"/>	<input checked="" type="radio"/>
Higher order functions are specific to OCaml.	<input type="radio"/>	<input checked="" type="radio"/>
Property based testing cannot be used in conjunction with unit testing.	<input type="radio"/>	<input checked="" type="radio"/>
In OCaml, anywhere we use map, we could use fold instead.	<input checked="" type="radio"/>	<input type="radio"/>
If $A <: B$ and $B <: C$ then $A <: C$	<input checked="" type="radio"/>	<input type="radio"/>
Every CFG can be represented by an NFA	<input type="radio"/>	<input checked="" type="radio"/>
In Rust, every variable is immutable by default	<input checked="" type="radio"/>	<input type="radio"/>
OCaml is dynamically typed because it uses type inference	<input type="radio"/>	<input checked="" type="radio"/>
Regular expressions are necessary to code a parser.	<input type="radio"/>	<input checked="" type="radio"/>
Safe Rust will never produce runtime errors.	<input type="radio"/>	<input checked="" type="radio"/>
By fixing all logic bugs, you have a secure program.	<input type="radio"/>	<input checked="" type="radio"/>
Higher order functions are specific to OCaml.	<input type="radio"/>	<input checked="" type="radio"/>
Property based testing cannot be used in conjunction with unit testing.	<input type="radio"/>	<input checked="" type="radio"/>
In OCaml, anywhere we use map, we could use fold instead.	<input checked="" type="radio"/>	<input type="radio"/>
Safe Rust will never be the source of a buffer overflow attack	<input checked="" type="radio"/>	<input type="radio"/>
If $A <: B$ and $C <: A$ then $A <: C$	<input type="radio"/>	<input checked="" type="radio"/>
Rust and OCaml are both statically typed	<input checked="" type="radio"/>	<input type="radio"/>
A parser will not accept meaningless programs	<input type="radio"/>	<input checked="" type="radio"/>

Every Regular Expression has an equivalent DFA

☒ T ☐ F

A Context Free Grammar could describe strings of length 6 which include balanced parenthesis

☒ T ☐ F

When writing an evaluator, the specification of the grammar is more important than the operational semantics

☐ T ☒ F

Regular Expressions can only be found in OCaml

☐ T ☒ F

You could theoretically implement project 4 in lambda calculus

☒ T ☐ F

$(\lambda x. x x)(\lambda x. x x)$  can be beta reduced to  $(\lambda x. x x)$

☐ T ☒ F

### Problem 2: Regex

[Total 5 pts]

Which of the following strings are accepted by the regular expression below?

$$\lambda^* \delta \sigma | [\omega \beta] \{2\}$$

Select NONE if none of the first five (5) options match.

- ☒ A  $\delta \sigma$     ☐ B  $\omega \beta \omega \beta$     ☐ C  $\lambda \lambda \sigma \beta \beta$     ☒ D  $\omega \beta$     ☐ E  $\delta \omega \lambda$     ☐ F NONE

### Problem 3: Regex

[Total 5 pts]

Which of the following strings are accepted by the regular expression below?

$$\lambda^* \delta \sigma | [\omega \beta] \{2\}$$

Select NONE if none of the first five (5) options match.

- ☒ A  $\delta \sigma$     ☐ B  $\omega \beta \omega \beta$     ☐ C  $\lambda \lambda \sigma \beta \beta$     ☒ D  $\omega \beta$     ☐ E  $\delta \omega \lambda$     ☐ F NONE

### Problem 4: Regex

[Total 5 pts]

Which of the following strings are accepted by the regular expression below?

$$\lambda^* \delta \sigma ? [\omega \beta] \{2\}$$

Select NONE if none of the first five (5) options match.

- ☒ A  $\lambda \lambda \sigma \beta \beta$     ☐ B  $\omega \beta \omega \beta$     ☐ C  $\delta \sigma$     ☒ D  $\omega \beta$     ☐ E  $\delta \omega \lambda$     ☐ F NONE

## Problem 5: Property Based Testing

[Total 6 pts]

Consider the following incorrect `mapip` function for a list of `i32`s in Rust. It is meant to apply the function to each `i32` element in a `Vec`, modifying the input `Vec` and not returning anything new.

```
fn mapip<F: Fn(i32) -> i32>(f: F, v: &mut Vec<i32>) -> Vec<i32> {
    let mut r = v.clone();
    for mut item in &mut r {
        *item = f(*item);
    }
    r
}
```

Consider the following property  $p$  about the `mapip` function:

$p$  : A `Vec` should have different values before and after it is passed into `mapip`.

Using a **correct** implementation of `mapip`, this property  $p$  should hold true for all valid inputs?

☒ Yes ☐ No

Using **our** implementation of `mapip`, this property  $p$  should hold true for all valid inputs?

☒ Yes ☐ No

Suppose I encode this property in Rust as the following:

```
#[test]
fn test<F: Fn(i32) -> i32>(f: F, v: Vec<i32>) {
    let initial = v.clone();
    mapip(f,v);
    assert_eq!(v, initial);
}
```

The above test function is a valid encoding of the property  $p$ .

☐ Yes ☒ No

## Problem 6: Property Based Testing

[Total 6 pts]

Consider the following incorrect `mapip` function for a list of `i32`s in Rust. It is meant to apply the function to each `i32` element in a `Vec`, modifying the input `Vec` and not returning anything new.

```
fn mapip<F: Fn(i32) -> i32>(f: F, v: &mut Vec<i32>) -> Vec<i32> {
    let mut r = v.clone();
    for mut item in &mut r {
        *item = f(*item);
    }
    r
}
```

Consider the following property  $p$  about the `mapip` function:

$p$  : A `Vec` should have different values before and after it is passed into `mapip`.

Using a **correct** implementation of `mapip`, this property  $p$  should hold true for all valid inputs?

☒ Yes ☐ No

Using **our** implementation of `mapip`, this property  $p$  should hold true for all valid inputs?

☒ Yes ☐ No

Suppose I encode this property in Rust as the following:

```
#[test]
fn test<F: Fn(i32) -> i32>(f: F, v: Vec<i32>) {
    let initial = v.clone();
    mapip(f,v);
    assert_eq!(v, initial);
}
```

The above test function is a valid encoding of the property  $p$ .

☐ Yes ☒ No

## Problem 7: Property Based Testing

[Total 6 pts]

Consider the following incorrect `mapip` function for a list of `i32`s in Rust. It is meant to apply the function to each `i32` element in a `Vec`, modifying the input `Vec` and not returning anything new.

```
fn mapip<F: Fn(i32) -> i32>(f: F, v: &mut Vec<i32>) -> Vec<i32> {
    let mut r = v.clone();
    for mut item in &mut r {
        *item = f(*item);
    }
    r
}
```

Consider the following property  $p$  about the `mapip` function:

$p$  : A `Vec` should have different values before and after it is passed into `mapip`.

Using a **correct** implementation of `mapip`, this property  $p$  should hold true for all valid inputs?

☒ Yes ☐ No

Using **our** implementation of `mapip`, this property  $p$  should hold true for all valid inputs?

☒ Yes ☐ No

Suppose I encode this property in Rust as the following:

```
#[test]
fn test<F: Fn(i32) -> i32>(f: F, v: Vec<i32>) {
    let initial = v.clone();
    mapip(f,v);
    assert_eq!(v, initial);
}
```

The above test function is a valid encoding of the property  $p$ .

☐ Yes

☒ No

## Problem 8: CFG Creation

[Total 4 pts]

Which Context Free Grammar(s) are equivalent to the regex:  $c^*(ab)^+d$ ?

Select all that apply.

- |   |  |
|---|--|
| <p>(A) <math>U \rightarrow MUD abU</math><br/><math>M \rightarrow cM \epsilon</math><br/><math>D \rightarrow d \epsilon</math></p>                                    | <p>(B) <math>U \rightarrow MDC</math><br/><math>D \rightarrow abD ab</math><br/><math>M \rightarrow cM \epsilon</math><br/><math>C \rightarrow d \epsilon</math></p> |
| <p>(C) <math>U \rightarrow MDC</math><br/><math>D \rightarrow abD ab</math><br/><math>M \rightarrow cM \epsilon</math><br/><math>C \rightarrow dC \epsilon</math></p> | <p>(D) <math>U \rightarrow MUD abU</math><br/><math>M \rightarrow c M \epsilon</math><br/><math>D \rightarrow d \epsilon</math></p>                                  |



## Problem 9: CFG Creation

[Total 4 pts]

Which Context Free Grammar(s) are equivalent to the regex:  $c^*(ab)^+d$ ?

Select all that apply.

- ☒ B 
$$\begin{aligned} U &\rightarrow MDC \\ D &\rightarrow abD|ab \\ M &\rightarrow cM|\epsilon \\ C &\rightarrow d|\epsilon \end{aligned}$$
- ☐ A 
$$\begin{aligned} U &\rightarrow MUD|abU \\ M &\rightarrow cM|\epsilon \\ D &\rightarrow d|\epsilon \end{aligned}$$
- ☐ D 
$$\begin{aligned} U &\rightarrow MUD|abU \\ M &\rightarrow c|M|\epsilon \\ D &\rightarrow d|\epsilon \end{aligned}$$
- ☐ C 
$$\begin{aligned} U &\rightarrow MDC \\ D &\rightarrow abD|ab \\ M &\rightarrow cM|\epsilon \\ C &\rightarrow dC|\epsilon \end{aligned}$$

### Problem 10: CFG Creation

[Total 4 pts]

Which Context Free Grammar(s) are equivalent to the regex:  $[ab]^+(c?d)^*$

Select all that apply.

☒ A  $S \rightarrow abS|abT$   
 $T \rightarrow cdT|dT|\epsilon$

☒ B  $S \rightarrow aS|bS|aT|bT$   
 $T \rightarrow dT|cdT|\epsilon$

☒ C  $S \rightarrow TF$   
 $T \rightarrow aT|bT|a|b$   
 $F \rightarrow UF|\epsilon$   
 $U \rightarrow cd|d$

☐ D  $S \rightarrow abS|abT|T$   
 $T \rightarrow cdT|dT|\epsilon$

## Problem 11: OCaml and Rust Typing

[Total 6 pts]

Give the type of the expression. If there is a type error, put "ERROR"

```
(* OCaml *)
fun x ->
  let (a,b) = x in
  fun y ->
    let a = (a+1, b > true) in
    (a::[y])
```

```
// Rust
{
  let a = if false {
    true > false
  } else {
    false
  };
  let b = true;
  (a, b)
}
```

$int * bool \Rightarrow int * bool \Rightarrow (int * bool)list$

$(bool, bool)$

## Problem 12: OCaml and Rust Evaluation

[Total 4 pts]

Evaluate the following expressions. If there is a compilation error, put "ERROR"

```
(* OCaml *)
let rec f x = match x with
  [] -> 3
  |x::xs -> List.fold_left x (f xs) [1;2;3] in

f [(fun a b -> a + b)]
```

```
// Rust
fn f1(x: i32, y: i32) -> i32 {
  x + y
}

fn f2(x: i32, y: i32) -> i32 {
  x * y
}

...
{
  let mut x = vec![-4, -2, 3];
  let mut a = true;
  for i in x.iter_mut() {
    if a {
      *i = f1(*i, *i);
      a = false;
    } else {
      *i = f2(*i, *i);
      a = true;
    }
  }
  x
}
```

9

$[-8, 4, 6]$

## Problem 13: OCaml and Rust Typing

[Total 6 pts]

Give the type of the expression. If there is a type error, put "ERROR"

```
// Rust
{
  let a = if false {
    true > false
  } else {
    false
  };
  let b = true;
  (a, b)
}
```

```
(* Ocaml *)
fun x ->
  let (a,b) = x in
  fun y ->
    let a = (a+1, b > true) in
    (a::[y])
```

(bool, bool)

$int * bool \Rightarrow int * bool \Rightarrow (int * bool)list$

## Problem 14: Ocaml and Rust Evaluation

[Total 4 pts]

Evaluate the following expressions. If there is a compilation error, put "ERROR"

```
(* Ocaml *)
let rec f x = match x with
  [] -> 3
|x::xs -> List.fold_left x (f xs) [1;2;3] in

f [(fun a b -> a + b)]
```

```
// Rust
fn f1(x: i32, y: i32) -> i32 {
  x + y
}

fn f2(x: i32, y: i32) -> i32 {
  x * y
}
...
{
  let mut x = vec![-4, -2, 3];
  let mut a = true;
  for i in x.iter_mut() {
    if a {
      *i = f1(*i, *i);
      a = false;
    } else {
      *i = f2(*i, *i);
      a = true;
    }
  }
  x
}
```

9

[-8, 4, 6]

## Problem 15: OCaml and Rust Typing

[Total 6 pts]

Give the type of the expression. If there is a type error, put "ERROR"

```
(* Ocaml *)
fun x ->
  let y = match x with
    [] -> x
    |(a,b)::xs -> (b,a)::xs
  in y
```

```
// Rust
{
  let x = true;
  let a = if x || false {
    4 + 2
  } else {
    false
  };
  let b = 6;
  a == b
}
```



## Problem 16: Ocaml and Rust Evaluation

[Total 4 pts]

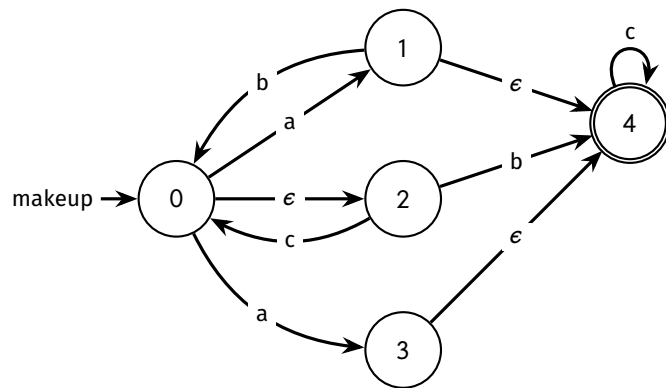
Evaluate the following expressions. If there is a compilation error, put "ERROR"

```
(* Ocaml *)
let rec f x = match x with
  [] -> []
  |x::xs ->
    (List.map (fun a -> a::[x]) [x])@f xs in
f [(fun a b -> a + b)]
```

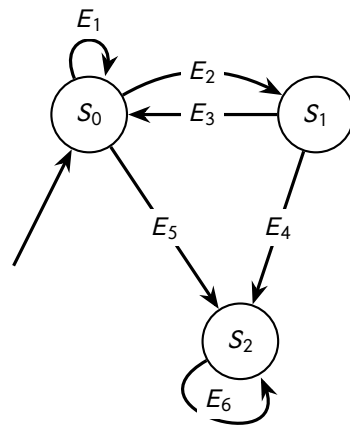
```
// Rust
fn f1(x: i32, y: i32) -> bool {
  x == y % 2
}
...
{
  let mut x = vec![(1,3),(2,4),(0,5)];
  let mut res = vec![];
  for i in x.into_iter() {
    match i{
      (a,b)=>res.push(f1(a,b))
    }
  };
  res
}
```

## Problem 17: NFA to DFA

[Total 10 pts]



Convert the above NFA to the below DFA. You **MUST** show your work to get any credit. (You will need to remove the Garbage state)



Scratch Space:

$S_0$ :	0,3,4	$S_1$ :	1,2,3,4	$S_2$ :	4
$E_1$ :	a	$E_2$ :	b	$E_3$ :	c
$E_4$ :	a	$E_5$ :	d	$E_6$ :	a

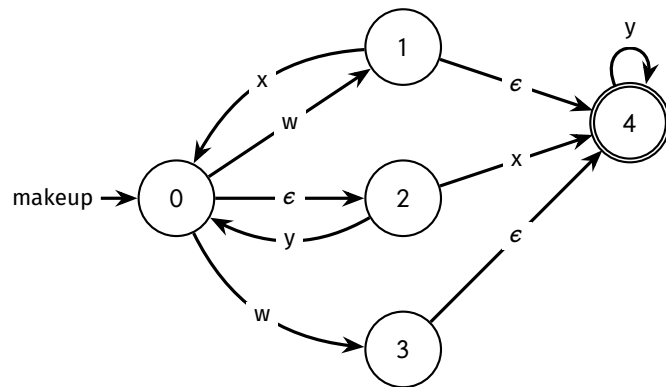
(a) Which states are the final (accepting) states? Select all that apply

[2 pts]

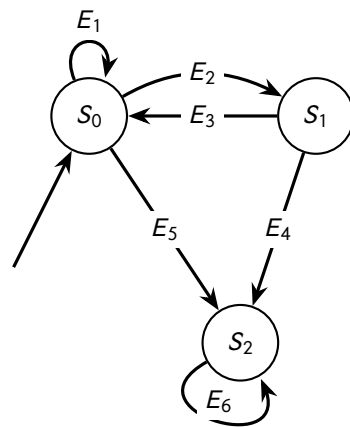
☒ State  $S_0$  ☒ State  $S_1$  ☒ State  $S_2$

## Problem 18: NFA to DFA

[Total 10 pts]



Convert the above NFA to the below DFA. You **MUST** show your work to get any credit. (You will need to remove the Garbage state)



Scratch Space:

$S_0$ :	<input type="text" value="0,3,4"/>	$S_1$ :	<input type="text" value="1,2,3,4"/>	$S_2$ :	<input type="text" value="4"/>
$E_1$ :	<input type="text" value="w"/>	$E_2$ :	<input type="text" value="x"/>	$E_3$ :	<input type="text" value="y"/>
$E_4$ :	<input type="text" value="w"/>	$E_5$ :	<input type="text" value="z"/>	$E_6$ :	<input type="text" value="w"/>

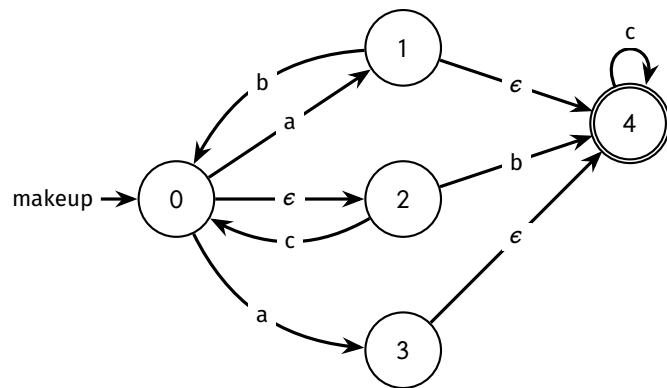
(a) Which states are the final (accepting) states? Select all that apply

[2 pts]

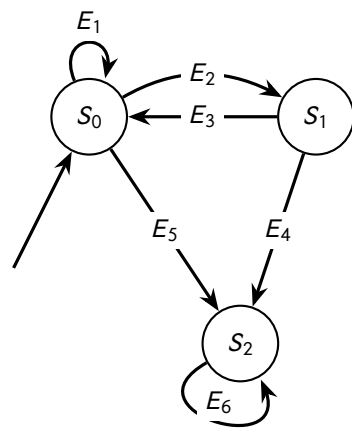
☒ A State  $S_0$  ☒ B State  $S_1$  ☒ C State  $S_2$

**Problem 19: NFA to DFA**

[Total 10 pts]



Convert the above NFA to the below DFA. You **MUST** show your work to get any credit. (You will need to remove the Garbage state)



Scratch Space:

$S_0$ :	<input type="text"/>	$S_1$ :	<input type="text"/>	$S_2$ :	<input type="text"/>
$E_1$ :	<input type="text"/>	$E_2$ :	<input type="text"/>	$E_3$ :	<input type="text"/>
$E_4$ :	<input type="text"/>	$E_5$ :	<input type="text"/>	$E_6$ :	<input type="text"/>

(a) Which states are the final (accepting) states? Select all that apply

[2 pts]

- ☒ A State  $S_0$    ☒ B State  $S_1$    ☒ C State  $S_2$



## Problem 20: Lexing, Parsing, Interpreting

[Total 10 pts]

Given the following CFG, and assuming the **Ocaml** type system and semantics, at what stage of language processing would each expression **fail**? Mark '**Valid**' if the expression would be accepted by the grammar and evaluate successfully. Assume the only symbols allowed are those found in the grammar.

$$\begin{aligned} E &\rightarrow \text{let } S = E \text{ in } E \mid \text{let ref } S = E ; E \mid R \\ R &\rightarrow R <> R \mid S := R \mid M \\ M &\rightarrow M * M \mid M - M \mid S \\ S &\rightarrow 1 \mid 2 \mid 3 \mid \text{true} \mid \text{false} \mid x \mid (E) \end{aligned}$$

For all  $x \in S$ ,  $x$  is a lowercase English character.

## Problem 21: Lexing, Parsing, Interpreting

[Total 10 pts]

Given the following CFG, and assuming the **Ocaml** type system and semantics, at what stage of language processing would each expression **fail**? Mark '**Valid**' if the expression would be accepted by the grammar and evaluate successfully. Assume the only symbols allowed are those found in the grammar.

$$\begin{aligned} E &\rightarrow \text{let } S = E \text{ in } E \mid \text{let ref } S = E ; E \mid R \\ R &\rightarrow R <> R \mid S := R \mid M \\ M &\rightarrow M * M \mid M - M \mid S \\ S &\rightarrow 1 \mid 2 \mid 3 \mid \text{true} \mid \text{false} \mid x \mid (E) \end{aligned}$$

For all  $x \in S$ ,  $x$  is a lowercase English character.

## Problem 22: Lexing, Parsing, Interpreting

[Total 10 pts]

Given the following CFG, and assuming the **Ocaml** type system and semantics, at what stage of language processing would each expression **fail**? Mark '**Valid**' if the expression would be accepted by the grammar and evaluate successfully. Assume the only symbols allowed are those found in the grammar.

$$\begin{aligned} E &\rightarrow \text{let } S = E \text{ in } E \mid \text{let ref } S = E ; E \mid R \\ R &\rightarrow R <> R \mid S := R \mid M \\ M &\rightarrow M * M \mid M - M \mid S \\ S &\rightarrow 1 \mid 2 \mid 3 \mid \text{true} \mid \text{false} \mid x \mid (E) \end{aligned}$$

For all  $x \in S$ ,  $x$  is a lowercase English character.

Lexer Parser Evaluator Valid

let x = x <> false in x	<input type="radio"/> L	<input type="radio"/> P	<input checked="" type="radio"/> E	<input type="radio"/> V
true := 3	<input type="radio"/> L	<input type="radio"/> P	<input checked="" type="radio"/> E	<input type="radio"/> V
let 2 - 1 = 1 in 3	<input type="radio"/> L	<input checked="" type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V
let ref a = 1 * 4; a := 5	<input checked="" type="radio"/> L	<input type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V
if true then false else 3	<input checked="" type="radio"/> L	<input type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V

Lexer Parser Evaluator Valid

let 2 - 1 = 1 in 3	<input type="radio"/> L	<input checked="" type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V
if true then false else 3	<input checked="" type="radio"/> L	<input type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V
let ref a = 1 * 4; a := 5	<input checked="" type="radio"/> L	<input type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V
true := 3	<input type="radio"/> L	<input type="radio"/> P	<input checked="" type="radio"/> E	<input type="radio"/> V
let x = x <> false in x	<input type="radio"/> L	<input type="radio"/> P	<input checked="" type="radio"/> E	<input type="radio"/> V

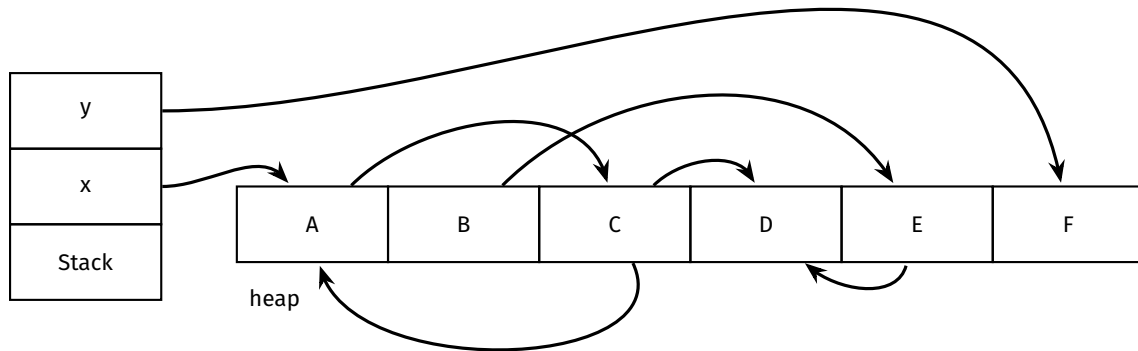
Lexer Parser Evaluator Valid

1 * 2 * 3	<input type="radio"/> L	<input type="radio"/> P	<input type="radio"/> E	<input checked="" type="radio"/> V
let x = (1 != 2) in x	<input checked="" type="radio"/> L	<input type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V
a := let x = 1 in 1	<input type="radio"/> L	<input checked="" type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V
let a = true in a - 1	<input type="radio"/> L	<input type="radio"/> P	<input checked="" type="radio"/> E	<input type="radio"/> V
((true))	<input type="radio"/> L	<input checked="" type="radio"/> P	<input type="radio"/> E	<input type="radio"/> V

## Problem 23: Garbage Collection

[Total 6 pts]

Given the following memory diagram:



(a) Select each piece of memory that should be marked as freed after calling Mark and Sweep at the time of this diagram. [3 pts]

Free A ☒ A Free B ☐ B Free C ☒ C Free D ☐ D Free E ☐ E Free F ☐ F None ☐ G

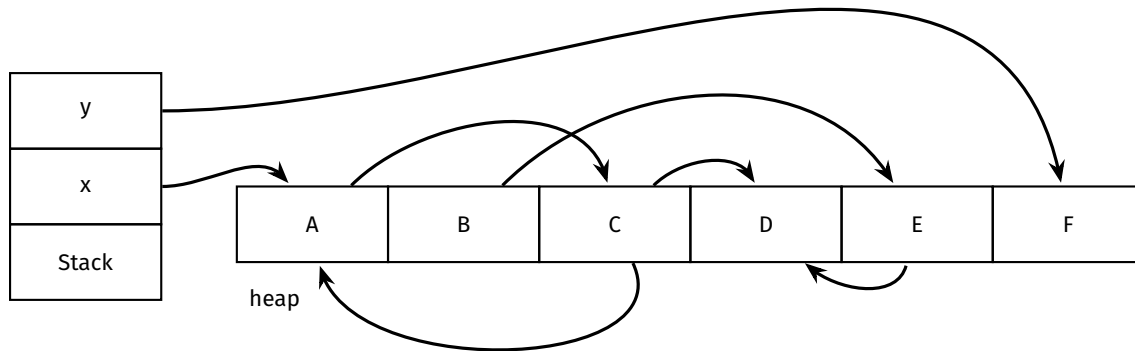
(b) Select each piece of memory that should be marked as freed at the time of this diagram using Reference Counting. [3 pts]

Free A ☐ A Free B ☐ B Free C ☐ C Free D ☐ D Free E ☐ E Free F ☐ F None ☒ G

## Problem 24: Garbage Collection

[Total 6 pts]

Given the following memory diagram:



(a) Select each piece of memory that should be marked as freed after calling Mark and Sweep at the time of this diagram. [3 pts]

Free A ☒ A Free B ☐ B Free C ☒ C Free D ☐ D Free E ☐ E Free F ☐ F None ☐ G

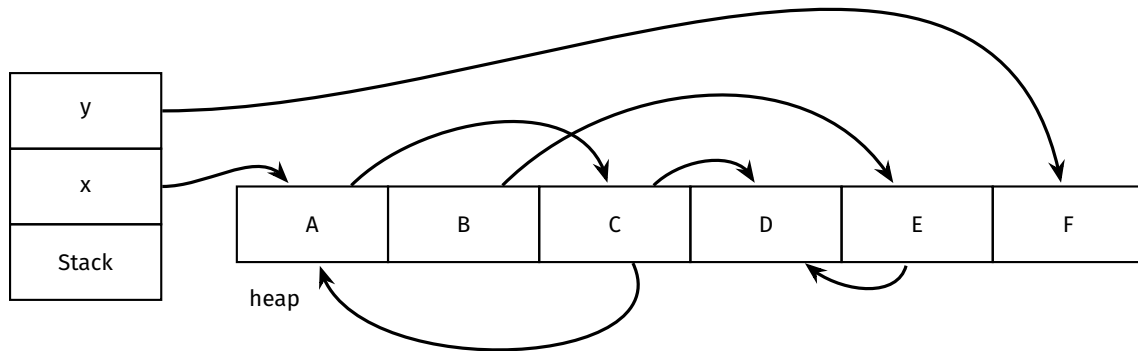
(b) Select each piece of memory that should be marked as freed at the time of this diagram using Reference Counting. [3 pts]

Free A ☐ A Free B ☐ B Free C ☐ C Free D ☐ D Free E ☐ E Free F ☐ F None ☒ G

## Problem 25: Garbage Collection

[Total 6 pts]

Given the following memory diagram:



(a) Select each piece of memory that should be marked as freed after calling Mark and Sweep at the time of this diagram. [3 pts]

Free A ☒ A Free B ☐ B Free C ☒ C Free D ☐ D Free E ☐ E Free F ☐ F None ☐ G

(b) Select each piece of memory that should be marked as freed at the time of this diagram using Reference Counting. [3 pts]

Free A ☐ A Free B ☐ B Free C ☐ C Free D ☐ D Free E ☐ E Free F ☐ F None ☒ G

## Problem 26: Lambda Calculus

[Total 8 pts]

(a) Reduce

[3 pts]

Order the following statements to produce a correct reduction of the following expression. The reduction you choose MUST use **lazy evaluation**. Write the letter corresponding to each step in the boxes on the right. **Note that you may not need to fill every box.**

$$(\lambda c. a b (c a)) ((\lambda a. c) (\lambda c. (\lambda a. b)))$$

A.  $c a$

B.  $a b (\lambda a. (\lambda c. (\lambda a. b)) a)$

B

C.  $a b ((\lambda a. c) (\lambda c. (\lambda a. b)) a)$

D.  $((\lambda a. c) (\lambda c. (\lambda a. b))) a$

F

E.  $a b (c a)$

F.  $a b (\lambda c. (\lambda a. b))$

G.  $(\lambda c. a b (c a)) ((\lambda c. (\lambda a. b)))$

H.  $(\lambda c. a b (c a)) c$

I.  $(a b ((\lambda a. c) a)) (\lambda a. b)$

J.  $(a b ((\lambda a. c) a)) (\lambda c. (\lambda a. b))$

(b) Free Variables:

[3 pts]

Circle the free variables in the expression below:

$$(\lambda a. (\lambda b. \textcircled{c}) (\lambda c. c) (\textcircled{b} \textcircled{b} (\lambda c. \textcircled{b})))$$

(c) Alpha Equivalence:

[2 pts]

Which of the following are alpha equivalent to the following expression (from part b):  $(\lambda a. (\lambda b. \textcircled{c}) (\lambda c. c) (\textcircled{b} \textcircled{b} (\lambda c. \textcircled{b})))$ ?

**Select all that apply.**

☒ A.  $(\lambda a. (\lambda a. c) (\lambda a. a) (b b (\lambda a. b)))$

☐ B.  $(\lambda b. (\lambda c. a) (\lambda a. a) (c c (\lambda a. c)))$

☒ C.  $(\lambda d. (\lambda f. c) (\lambda g. g) (b b (\lambda h. b)))$

☐ D.  $(\lambda c. (\lambda a. c) (\lambda b. b) (b b (\lambda a. b)))$

## Problem 27: Lambda Calculus

[Total 8 pts]

(a) Reduce

[3 pts]

Order the following statements to produce a correct reduction of the following expression. The reduction you choose **MUST** use **eager evaluation**. Write the letter corresponding to each step in the boxes on the right. **Note that you may not need to fill every box.**

$$(\lambda f. d \ e \ (f \ d)) \ ((\lambda d. f) \ (\lambda f. (\lambda d. e)))$$

A.  $f \ d$

B.  $d \ e \ (\lambda d. (\lambda f. (\lambda d. e)) \ d)$

D

C.  $d \ e \ ((\lambda d. f) \ (\lambda f. (\lambda d. e)) \ d)$

D.  $((\lambda d. f) \ (\lambda f. (\lambda d. e))) \ d$

F

E.  $d \ e \ (f \ d)$

F.  $d \ e \ (\lambda f. (\lambda d. e))$

G.  $(\lambda f. d \ e \ (f \ d)) \ ((\lambda f. (\lambda d. e)))$

H.  $(\lambda f. d \ e \ (f \ d)) \ f$

I.  $(d \ e \ ((\lambda d. f) \ d)) \ (\lambda d. e)$

J.  $(d \ e \ ((\lambda d. f) \ d)) \ (\lambda f. (\lambda d. e))$

(b) Free Variables:

[3 pts]

Circle the free variables in the expression below:

$$(\lambda d. (\lambda e. \textcircled{f}) (\lambda f. f) (\textcircled{e} \ \textcircled{e} (\lambda f. \textcircled{e})))$$

(c) Alpha Equivalence:

[2 pts]

Which of the following are alpha equivalent to the following expression (from part b):  $(\lambda d. (\lambda e. \textcircled{f}) (\lambda f. f) (\textcircled{e} \ \textcircled{e} (\lambda f. \textcircled{e})))$ ?

**Select all that apply.**

- ☒ A.  $(\lambda d. (\lambda d. f) (\lambda d. d) (e \ e (\lambda d. e)))$
- ☐ B.  $(\lambda e. (\lambda f. d) (\lambda d. d) (f \ f (\lambda d. f)))$
- ☐ C.  $(\lambda d. (\lambda f. f) (\lambda g. g) (e \ e (\lambda h. e)))$
- ☐ D.  $(\lambda f. (\lambda d. f) (\lambda e. e) (e \ e (\lambda d. e)))$

## Problem 28: Lambda Calculus

[Total 8 pts]

(a) Reduce

[3 pts]

Order the following statements to produce a correct reduction of the following expression. The reduction you choose MUST use **eager evaluation**. Write the letter corresponding to each step in the boxes on the right. **Note that you may not need to fill every box.**

$$(\lambda m. j \ k \ (m \ j)) \ ((\lambda j. m) \ (\lambda m. (\lambda j. k)))$$

A.  $m \ j$

B.  $j \ k \ (\lambda j. (\lambda m. (\lambda j. k)) \ j)$

H

C.  $j \ k \ ((\lambda j. m) \ (\lambda m. (\lambda j. k)) \ j)$

D.  $((\lambda j. m) \ (\lambda m. (\lambda j. k))) \ j$

E

E.  $j \ k \ (m \ j)$

F.  $j \ k \ (\lambda m. (\lambda j. k))$

G.  $(\lambda m. j \ k \ (m \ j)) \ ((\lambda m. (\lambda j. k)))$

H.  $(\lambda m. j \ k \ (m \ j)) \ m$

I.  $(j \ k \ ((\lambda j. m) \ j)) \ (\lambda j. k)$

J.  $(j \ k \ ((\lambda j. m) \ j)) \ (\lambda m. (\lambda j. k))$

(b) Free Variables:

[3 pts]

Circle the free variables in the expression below:

$$(\lambda j. (\lambda k. \textcircled{m}) (\lambda m. m) (\textcircled{k} \ \textcircled{k} (\lambda m. \textcircled{k}))))$$

(c) Alpha Equivalence:

[2 pts]

Which of the following are alpha equivalent to the following expression (from part b):  $(\lambda j. (\lambda k. \textcircled{m}) (\lambda m. m) (\textcircled{k} \ \textcircled{k} (\lambda m. \textcircled{k}))))$ ? **Select all that apply.**

- ☒ A.  $(\lambda j. (\lambda j. m) (\lambda j. j) (k \ k (\lambda j. k)))$
- ☐ B.  $(\lambda k. (\lambda m. j) (\lambda j. j) (m \ m (\lambda j. m)))$
- ☐ C.  $(\lambda d. (\lambda f. m) (\lambda g. g) (k \ k (\lambda h. k)))$
- ☐ D.  $(\lambda m. (\lambda j. m) (\lambda k. k) (k \ k (\lambda j. k)))$

## Problem 29: Operational Semantics

[Total 8 pts]

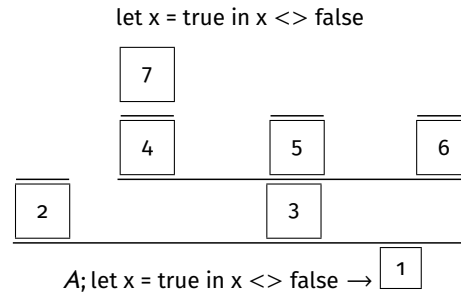
Consider the following rules of OCaml:

$$\frac{}{A; true \rightarrow true} \quad \frac{}{A; false \rightarrow false} \quad \frac{}{A; n \rightarrow n} \quad \frac{A(x) = v}{A; x \rightarrow v}$$

$$\frac{A; e_1 \rightarrow v_1 \quad A, x : v_1; e_2 \rightarrow v_2}{A; let x = e_1 in e_2 \rightarrow v_2} \quad \frac{A; e_1 \rightarrow v_1 \quad A; e_2 \rightarrow v_2 \quad v_3 \text{ is } v_1 <> v_2}{A; e_1 <> e_2 \rightarrow v_3}$$

Using OCaml as the metalanguage, prove the following sentence evaluates.

**IMPORTANT: you must fill in the blanks to receive credit.**



Scratch Space:

Blank 1: *true*

Blank 2: *A; true  $\rightarrow$  true*

Blank 3: *A, x : true; x <> false  $\rightarrow$  true*

Blank 4: *A, x : true; x  $\rightarrow$  true*

Blank 5: *A, x : true; false  $\rightarrow$  false*

Blank 6: *true is true <> false*

Blank 7: *A, x : true(x) = true*



## Problem 30: Operational Semantics

[Total 8 pts]

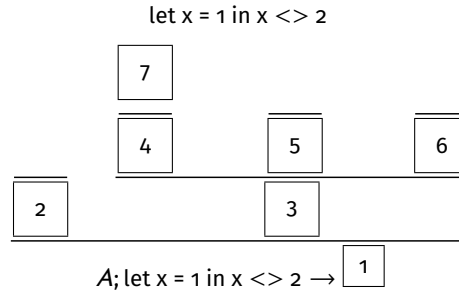
Consider the following rules of OCaml:

$$\frac{}{A; true \rightarrow true} \quad \frac{}{A; false \rightarrow false} \quad \frac{}{A; n \rightarrow n} \quad \frac{A(x) = v}{A; x \rightarrow v}$$

$$\frac{A; e_1 \rightarrow v_1 \quad A, x : v_1; e_2 \rightarrow v_2}{A; let\ x = e_1\ in\ e_2 \rightarrow v_2} \quad \frac{A; e_1 \rightarrow v_1 \quad A; e_2 \rightarrow v_2 \quad v_1\ is\ v_1\ <>\ v_2}{A; e_1\ <>\ e_2 \rightarrow v_3}$$

Using OCaml as the metalanguage, prove the following sentence evaluates.

**IMPORTANT: you must fill in the blanks to receive credit.**



Scratch Space:

Blank 1: 1

Blank 2: A; 1 -> 1

Blank 3: A, x : 1; x <> 2 -> true

Blank 4: A, x : 1; x -> 1

Blank 5: A, x : 1; 2 -> 2

Blank 6: true is 1 <> 2

Blank 7: A, x : 1(x) = 1

## Problem 31: Operational Semantics

[Total 8 pts]

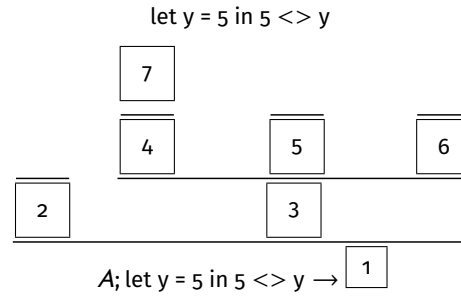
Consider the following rules of OCaml:

$$\frac{}{A; true \rightarrow true} \quad \frac{}{A; false \rightarrow false} \quad \frac{}{A; n \rightarrow n} \quad \frac{A(x) = v}{A; x \rightarrow v}$$

$$\frac{A; e_1 \rightarrow v_1 \quad A, x : v_1; e_2 \rightarrow v_2}{A; \text{let } x = e_1 \text{ in } e_2 \rightarrow v_2} \quad \frac{A; e_1 \rightarrow v_1 \quad A; e_2 \rightarrow v_2 \quad v_3 \text{ is } v_1 <> v_2}{A; e_1 <> e_2 \rightarrow v_3}$$

Using OCaml as the metalanguage, prove the following sentence evaluates.

**IMPORTANT: you must fill in the blanks to receive credit.**



Scratch Space:

Blank 1: bool

Blank 2:  $G \vdash 5 : int$

Blank 3:  $G, x : int \vdash x <> 7 : bool$

Blank 4:  $G, x : int \vdash x : int$

Blank 5:  $G, x : int \vdash 7 : int$

Blank 6:  $G, x : int(x) = int$

Blank 7:

## Problem 32: Ownership and Lifetimes

[Total 8 pts]

Does the code compile? ☒ Yes ☐ No

```
1 fn main(){
2   let mut x = String::from("hello");
3   let y = &x;
4   println!("{}",x,y);
5 }
```

If **no**, explain why not in one sentence:

Does the code compile? ☐ Yes ☒ No

```
1 fn main(){
2   let x = String::from("Hello");
3   let mut y = &x;
4   y.push_str(" world");
5   println!("{}",x);
6 }
```

If **no**, explain why not in one sentence:

y is not borrowed as mutable. so cannot push\_str

Does the code compile? ☒ Yes ☐ No

```
1 fn main(){
2   let mut x = String::from("Hello");
3   let _a = &x;
4   let y = &mut x;
5   y.push_str(" world");
6   println!("{}",x);
7 }
```

If **no**, explain why not in one sentence:

```
1 fn function(s1: String,
2            s2: String,
3            f:bool)->usize{
4   if f {s1.len()} else{s2.len()}
5 }

6 fn main(){
7   let a = String::from("hello");
8   let b = a.clone();
9   let c = function(b,a,true);
10  println!("{}",a,c);
11 }
```

Does the code compile? ☐ Yes ☒ No

If **no**, explain why not in one sentence:

a is no longer owner after function call

## Problem 33: Ownership and Lifetimes

[Total 8 pts]

Does the code compile? ☒ Y Yes ☐ F No

```
1 fn main(){
2   let mut x = String::from("hello");
3   let y = &x;
4   println!("{}",x,y);
5 }
```

If **no**, explain why not in one sentence:

Does the code compile? ☐ Y Yes ☒ F No

```
1 fn main(){
2   let x = String::from("Hello");
3   let mut y = &x;
4   y.push_str(" world");
5   println!("{}",x);
6 }
```

If **no**, explain why not in one sentence:

y is not borrowed as mutable. so cannot pushstr

Does the code compile? ☒ Y Yes ☐ F No

```
1 fn main(){
2   let mut x = String::from("Hello");
3   let _a = &x;
4   let y = &mut x;
5   y.push_str(" world");
6   println!("{}",x);
7 }
```

If **no**, explain why not in one sentence:

```
1 fn function(s1: String,
2             s2: String,
3             f:bool)->usize{
4     if f {s1.len()} else{s2.len()}
5 }

6 fn main(){
7     let a = String::from("hello");
8     let b = a.clone();
9     let c = function(b,a,true);
10    println!("{}", has length {},a,c);
11 }
```

Does the code compile? ☐ Y Yes ☒ F No

If **no**, explain why not in one sentence:

a is no longer owner after function call

## Problem 34: Ownership and Lifetimes

[Total 8 pts]

Does the code compile? ☒ Y Yes ☐ F No

```
1 fn main(){
2   let x = String::from("Hello");
3   let mut y = &mut x;
4   y.push_str(" world");
5   println!("{}",x);
6 }
```

If **no**, explain why not in one sentence:

a is no longer owner after function call

Does the code compile? ☒ Y Yes ☐ F No

```
1 fn main(){
2   let mut x = String::from("hello");
3   let y = &mut x;
4   println!("{}",x,y);
5 }
```

If **no**, explain why not in one sentence:

```
1 fn function(s1: String,
2             s2: String,
3             f:bool)->usize{
4   if f {s1.len()} else{s2.len()}
5 }

6 fn main(){
7   let a = String::from("hello");
8   let b = a.clone();
9   let c = function(b,a,true);
10  println!("{}",a,c);
11 }
```

Does the code compile? ☐ Y Yes ☒ F No

If **no**, explain why not in one sentence:

y is not borrowed as mutable. so cannot pushstr

Does the code compile? ☒ Y Yes ☐ F No

```
1 fn main(){
2   let mut x = String::from("Hello");
3   let _a = &x;
4   let y = &mut x;
5   println!("{}",x);
6   y.push_str(" world");
7 }
```

If **no**, explain why not in one sentence:

## Problem 35: Ocaml Coding

[Total 10 pts]

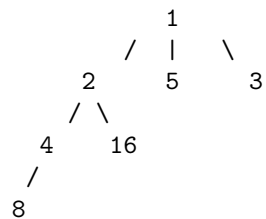
**Restrictions:** You are **not** allowed to use imperative OCaml; you can define recursive helper/helper functions below the function signatures we provided. You are not allowed to use any List module functions except the ones already given to you on the cheat sheet, otherwise you may use anything in StdLib.

Write a function called `largestsumpath` that, given a tree, returns a list of the path from the root to the node in the tree that gives the largest sum when each value on the path is added up.

If there are multiple tied sums, you can return any one path.  
Return an empty list if the tree is empty.

```
type 'a tree = Petal | Stem of 'a * 'a tree list
```

Example tree `t`:



```
let t = Stem (1,
  [Stem (2,
    [Stem (4,
      [Stem (8, [Petal])
    ]);
    Stem (6, [Petal])
  ]);
  Stem (3, [Petal]);
  Stem (5, [Petal])
])
```

Expected Output:

```
largest_sum_path t = [1;2;16]
```

Write your code on the next page.

```

let rec largest_sum_path t =
  match t with
  | Petal -> []
  | Stem(curr, children) ->
    match (List.map (fun x -> curr::(largest_path x)) children) with
    [] -> failwith "not possible"
  | x::xs -> List.fold_left (fun a x -> if List.length x > List.length a then x else a) x xs

```

### Problem 36: Ocaml Coding

[Total 10 pts]

**Restrictions:** You are **not** allowed to use imperative OCaml; you can define recursive helper/helper functions below the function signatures we provided. You are not allowed to use any List module functions except the ones already given to you on the cheat sheet, otherwise you may use anything in StdLib.

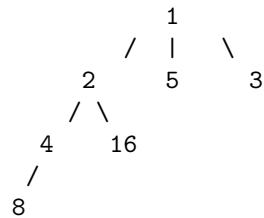
Write a function called `largestsumpath` that, given a tree, returns a list of the path from the root to the node in the tree that gives the largest sum when each value on the path is added up.

If there are multiple tied sums, you can return any one path.

Return an empty list if the tree is empty.

```
type 'a tree = External | Internal of 'a * 'a tree list
```

Example tree t:



```

let t = Internal (1,
  [Internal (2,
    [Internal (4,
      [Internal (8, [External])
    ]);
    Internal (6, [External])
  ]);
  Internal (3, [External]);
  Internal (5, [External])
])

```

Expected Output:

```
largest_sum_path t = [1;2;16]
```

Write your code on the next page.

```

let rec largest_sum_path t =
  match t with
  | Petal -> []
  | Stem(curr, children) ->
    match (List.map (fun x -> curr::(largest_path x)) children) with
    [] -> failwith "not possible"
  | x::xs -> List.fold_left (fun a x -> if List.length x > List.length a then x else a) x xs

```

### Problem 37: Ocaml Coding

[Total 10 pts]

**Restrictions:** You are **not** allowed to use imperative OCaml; you can define recursive helper/helper functions below the function signatures we provided. You are not allowed to use any List module functions except the ones already given to you on the cheat sheet, otherwise you may use anything in StdLib.

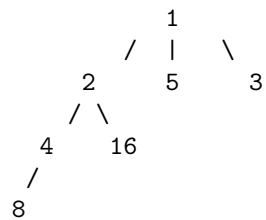
Write a function called `largestsumpath` that, given a tree, returns a list of the path from the root to the node in the tree that gives the largest sum when each value on the path is added up.

If there are multiple tied sums, you can return any one path.

Return an empty list if the tree is empty.

```
type int tree = Petal | Stem of int * int tree list
```

Example tree t:



```

let t = Stem (1,
  [Stem (2,
    [Stem (4,
      [Stem (8, [Petal])
    ]);
    Stem (16, [Petal])
  ]);
  Stem (3, [Petal]);
  Stem (5, [Petal])
])

```

Expected Output:

```
largest_sum_path t = [1;2;16]
```

Write your code on the next page.



```

let rec largest_sum_path t =
  match t with
  | Petal -> []
  | Stem(curr, children) ->
    match (List.map (fun x -> curr::(largest_path x)) children) with
    | [] -> failwith "not possible"
    | x::xs -> List.fold_left (fun a x -> if List.length x > List.length a then x else a) x xs

```

## Problem 38: Rust Coding

[Total 5 pts]

**Restrictions:** You may not use Rust's built in map or fold for this function.

Recall the higher order function `fold` in OCaml. We are going to write a version of it in Rust, though we are restricting the types of the input `vec` and accumulator to `u32`s rather than generics for simplicity.

`f` in the type signature is a function that takes in 2 `u32`s (one an element of the `Vec` and one an accumulator) and returns a `u32`.

Note: You can do `foldleft` or `foldright` but `foldleft` is probably easier.

### Example

```
fn add (e: u32, mut a : u32) -> u32 {
    a + e
}
```

```
fn main() {
    let mut v = vec![1,2,3];
    let mut a = 0;
    let x = fold (add, v, a);
    println!("{:?}", x); // prints 6
}
```

Write your code for fold below:

```
fn fold (f: impl Fn(u32, u32) -> u32, v: Vec<u32>, mut a: u32) -> u32 {
    for i in v {
        a = f(i, a);
    }
    a
}
```

```
}
```

## Problem 39: Rust Coding

[Total 5 pts]

**Restrictions:** You may not use Rust's built in map or fold for this function.

Recall the higher order function `fold` in OCaml. We are going to write a version of it in Rust, though we are restricting the types of the input `vec` and accumulator to `u32`s rather than generics for simplicity.

`f` in the type signature is a function that takes in 2 `u32`s (one an element of the `Vec` and one an accumulator) and returns a `u32`.

Note: You can do `foldleft` or `foldright` but `foldleft` is probably easier.

### Example

```
fn add (e: u32, mut a : u32) -> u32 {
    a + e
}
```

```
fn main() {
    let mut v = vec![1,2,3];
    let mut a = 0;
    let x = fold (add, v, a);
    println!("{:?}", x); // prints 6
}
```

Write your code for fold below:

```
fn fold (f: impl Fn(u32, u32) -> u32, v: Vec<u32>, mut a: u32) -> u32 {
    for i in v {
        a = f(i, a);
    }
    a
}
```

```
}
```

## Problem 40: Rust Coding

[Total 5 pts]

**Restrictions:** You may not use Rust's built in map or fold for this function.

Consider the higher order function `filter`. `filter` works by taking in a function and an list, and returning a new list of all the values in the list which cause the function to return true. We will modify this slightly. We will first restrict the types of the input list (vector here), to `u32`. Second, we want it to occur in place so we don't have to return a new vector.

`f` in the type signature is a function that takes in a `u32` (the element of the `Vec`) and returns a `bool`.

### Example

```
fn pos (e: u32) -> bool {
    e > 0
}

fn main() {
    let mut v = vec![1,-2,3];
    myfilter(pos,&mut v);
    println!("{:?}", v); // prints [1,3]
}
```

Write your code for `myfilter` below:

```
fn myfilter (f: impl Fn(u32) -> bool, v: Vec<u32>){
    for i in v {
        a = f(i, a);
    }
    a
}
```

```
}
```

### Problem 41: Extra Credit

[Total 2 pts]

For the following, you may only receive up to 2 points regardless of how many answers you give.

(a) Staff Stalking

[1 pts]

What is your discussion TA's name and what is your discussion's section number?

(b) Staff Stalking

[1 pts]

If your Discussion TA were an animal what would they be?

(c) Colon Parenthesis

[1 pts]

Write a poem!

### Problem 42: Extra Credit

[Total 2 pts]

For the following, you may only receive up to 2 points regardless of how many answers you give.

(a) Staff Stalking

[1 pts]

What is your discussion TA's name and what is your discussion's section number?

(b) Staff Stalking

[1 pts]

If your Discussion TA were an animal what would they be?

(c) Colon Parenthesis

[1 pts]

Write a poem!

### Problem 43: Extra Credit

[Total 2 pts]

For the following, you may only receive up to 2 points regardless of how many answers you give.

(a) Staff Stalking

[1 pts]

What is your discussion TA's name and what is your discussion's section number?

(b) Staff Stalking

[1 pts]

If your Discussion TA were an animal what would they be?

(c) Colon Parenthesis

[1 pts]

Write a poem!

**For Scratch Work - Do not tear off. Indicate the problem with your work**

# Cheat Sheet

## OCaml

```
(* Map and Fold *)
(* ('a -> 'b) -> 'a list -> 'b list *)
let rec map f l = match l with
  [] -> []
  |x::xs -> (f x)::(map f xs)

(* ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a *)
let rec fold_left f a l = match l with
  [] -> a
  |x::xs -> fold_left f (f a x) xs

(* ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b *)
let rec fold_right f l a = match l with
  [] -> a
  |x::xs -> f x (fold_right f xs a)

(* 'a list -> int *)
List.length: Returns the length
(number of elements) of the given list.
```

```
(* Regex in OCaml *)
Re.Posix.re: string -> regex
Re.compile: regex -> compiled_regex

Re.exec: compiled_regex -> string -> group
Re.exect: compiled_regex -> string -> bool
Re.exect_opt: compiled_regex -> string -> group option

Re.matches: compiled_regex -> string -> string list

Re.Group.get: group -> int -> string
Re.Group.get_opt: group -> int -> string option
```

```
(* OCaml Function Types *)
:: -: 'a -> 'a list -> 'a list

@ -: 'a list -> 'a list -> 'a list
```

## Structure of Regex

```
R  ->  Ø
    |  σ
    |  ε
    |  RR
    |  R|R
    |  R*
```

```
+, -, *, / -: int -> int -> int
+., -, *, /. -: float -> float -> float

&&, || -: bool -> bool -> bool
not -: bool -> bool

^ -: string -> string -> string

=>, >, =, <, <= :- 'a -> 'a -> bool
```

## Regex

*	zero or more repetitions of the preceding character or group
+	one or more repetitions of the preceding character or group
?	zero or one repetitions of the preceding character or group
.	any character
$r_1 r_2$	$r_1$ or $r_2$ (eg. $a b$ means 'a' or 'b')
[abc]	match any character in abc
[^ $r_1$ ]	anything except $r_1$ (eg. [^abc] is anything but an 'a', 'b', or 'c')
[ $r_1$ - $r_2$ ]	range specification (eg. [a-z] means any letter in the ASCII range of a-z)
{n}	exactly n repetitions of the preceding character or group
{n,}	at least n repetitions of the preceding character or group
{m,n}	at least m and at most n repetitions of the preceding character or group
^	start of string
\$	end of string
( $r_1$ )	capture the pattern $r_1$ and store it somewhere (match group in Python)
\d	any digit, same as [0-9]
\s	any space character like \n, \t, \r, \f, or space



## NFA to DFA Algorithm (Subset Construction Algorithm)

NFA (input):  $(\Sigma, Q, q_0, F_n, \delta)$ , DFA (output):  $(\Sigma, R, r_0, F_d, \delta_n)$

```

 $R \leftarrow \{\}$ 
 $r_0 \leftarrow \epsilon - \text{closure}(\sigma, q_0)$ 
while  $\exists$  an unmarked state  $r \in R$  do
    mark  $r$ 
    for all  $a \in \Sigma$  do
         $E \leftarrow \text{move}(\sigma, r, a)$ 
         $e \leftarrow \epsilon - \text{closure}(\sigma, E)$ 
        if  $e \notin R$  then
             $R \leftarrow R \cup \{e\}$ 
        end if
         $\sigma_n \leftarrow \sigma_n \cup \{r, a, e\}$ 
    end for
end while
 $F_d \leftarrow \{r \mid \exists s \in r \text{ with } s \in F_n\}$ 

```

## Rust

```

// Vectors
let vec = Vec::new(); // makes a new vector
let vec1 = vec![1,2,3]

vec.push(ele); // Pushes the element 'ele'
                // to end of the vector 'vec'

// Strings
let string = String::from("Hello");

string.push_str(&str); // appends the str
                       // to string

vec.to_iter(); // returns an iterator for vec

vec.iter_mut(); // returns an iterator that
                // allows modifying each value.

string.chars() // returns an iterator of chars
               // over the a string

iter.rev();    // reverses an iterators direction

iter.next();   // returns an Option of the next
               // item in the iterator.

struct Building{ // example of struct
    name:String,
    floors:i32,
    locationx:f32,
    locationy:f32,
}

enum Option<T>{ Some(T); None } //enum Option type
option.unwrap(); // returns the item in an Option or
                // panics if None

```