CMSC330 Spring 2019 Midterm 2
11:00am / 12:15pm / 2:00pm

Name (PRINT YOUR NAME as it appears on gradescope):

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Discussion Time (circle one) **10am 11am 12pm 1pm 2pm 3pm**

Instructions

- Do not start this test until you are told to do so!
- You have 75 minutes to take this midterm.
- This exam has a total of 100 points, so allocate 45 seconds for each point.
- This is a closed book exam. No notes or other aids are allowed.
- Answer essay questions concisely in 2-3 sentences. Longer answers are not needed.
- For partial credit, show all of your work and clearly indicate your answers.
- Write neatly. Credit cannot be given for illegible answers.

	Problem	Score
1	PL Concepts	/13
2	Finite Automata	/31
3	Context Free Grammars	/17
4	Parsing	/16
5	Operational Semantics	/10
6	Lambda Calculus	/13
	Total	/100

1. PL concepts [13 pts]

A) [5 pts] Circle true or false for each of the following 5 questions (1 point each)

True / False	In OCaml, if an exception is thrown, then the executing program will terminate
True / False	OCaml variables are immutable
True / False	If x and y are aliases, changing the content in the location referenced by x will cause it to no longer be an alias of y
True / False	If a lambda calculus expression reduces to a beta-normal form using call-by-value order, then it will also do so using call-by-name
True / False	You can create a cyclic data structure in OCaml (i.e., one that points to itself)

B) [4 pts] Consider the following OCaml definitions for `f`, `g`, and `h` (each is a `int -> int` function).

```
let f z =
  let y = ref 0 in
  y := !y + z;
  !y
```

```
let g =
  let x = ref 1 in
  (fun z ->
    x := !x + 1;
    !x+z)
```

```
let h =
  (fun z -> let x = z+1 in
    let _ = (print_int z,print_int x) in
    0)
```

Answer:

Which of these functions is not *referentially transparent*?	
Which function's execution outcome *depends on OCaml's evaluation order*	
What is a *side effect* carried out by at least one of the functions?	
Which function's execution is *only* interesting/useful because of its side effects, not what it returns?	

C) [4 pts] Check the box next to each function that is *tail recursive* (they all type check and run properly).

☐
```
let rec sum lst =
  match lst with
    [] -> 0
  | h::t-> h + sum t
```

☐
```
let rec max lst r =
  match lst with
    [] -> r
  | h::t ->
      if r>h then max t r
             else max t h
```

☐
```
let rec pow2 x =
  if x = 1 then true
  else
    let y = x/2 in
    if y*2 = x then pow2 y
    else false
```

☐
```
let rec prod lst =
  match lst with
    [] -> 1
  | h::t -> (prod t) * h
```

2. Finite Automata [31 pts]

A) [4 pts] Circle true or false for each of the following 4 questions (1 point each)

True / False NFAs are more expressive than DFAs (i.e., they can describe more languages)

True / False Every CFG has an equivalent NFA

True / False Every formal language has a unique DFA that generates it

True / False Regexes are more expressive (can generate more languages) than DFAs

B) [6 pts] For each of the following statements, check the DFA box if it's true for DFAs, and the NFA box for NFAs. You may check neither or both boxes.

☐DFA ☐NFA Can transition to multiple states at once with a symbol

☐DFA ☐NFA Can have epsilon transitions

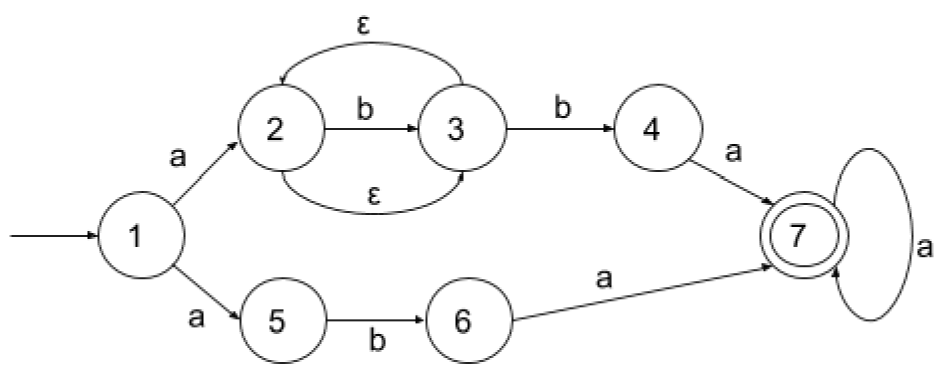
☐DFA ☐NFA Can have multiple final states

☐DFA ☐NFA Always has at least one final state

☐DFA ☐NFA Easy to translate directly from a regular expression

☐DFA ☐NFA Can accept an empty string

C) [6 pts] Draw a DFA that is equivalent to the following NFA.



D) [4 pts] Circle any of the following strings that would be accepted by the nfa from the previous problem.

aba **abbbbba** **aa** **abaa**

E) [6 pts] Draw an NFA that accepts the same language as the regex (**a***b**)|(**cd**). Here are some examples this NFA will accept: **b**, **ab**, **cd**, **aab**, **aaaaab**

F) [5 pts] Draw a DFA that accepts strings of the form $\mathbf{a}^n\mathbf{b}^n$ where $0 \le n \le 3$ over $\Sigma = \{ \mathbf{a},\mathbf{b} \}$

3. Context Free Grammars [17 pts]

A) [4 pts] Check the box next to the strings that are accepted by the following CFG. Note that here and below all nonterminals are in italics (like *T* and *W*) and terminals are in bold (like **a**, **b**).

T → **a***W* | **b**
W → **b** | **b***T* | **a***W*

☐ **abba** ☐ **aaabb** ☐ **baa** ☐ **aab**

B) [5 pts] Create a CFG for the language of all strings of the form $n^x f^z a^y$ where $x \geq y \geq 0$ and $z > 0$. Example strings in the language are **nfa**, **f**, **nnnfaa**. Example strings *not* in the language are **a**, **n**, **fa**, **nfaa**.

C) [4 pts] Rewrite the following grammar so that it can be parsed by a recursive descent parser. Note that parentheses and commas, below, are terminals (along with **r**, **u**, and **o**).

S → *A***)**
A → *A***,r** | *A***,u** | **(o**

D) [4 pts] The following CFG is ambiguous. Rewrite the grammar to remove the ambiguity. Note that minus sign is a terminal (along with **1**, **2**, and **3**).

E → *E* **-** *E* | *N*
N → **1** | **2** | **3**

4. Parsing and Scanning [16 pts]

A) [3 pts] Recall the scanner for SmallC. Suppose, when you tokenize the variable "**`for2`**", your tokenizer returned **`[Tok_ID("for");Tok_Int(2)]`** instead of **`[Tok_ID("for2")]`**. How would you fix this? (Write 1-2 sentences only.)

B) [5 pts] Consider the following CFG. Compute the first sets for each nonterminal.

$S \rightarrow mB \mid aA$
$A \rightarrow cS \mid \varepsilon$
$B \rightarrow 1\#S \mid dB \mid St \mid Ao$

FIRST(S) =

FIRST(A) =

FIRST(B) =

C) [8 pts] Complete the implementation for a recursive-descent parser for the provided CFG, given on the next page. Write your answer on the next page.

(scratch space, do not write your final answer here)

```
exception ParseError of string

let tok_list = ref [];;

let match_tok x = match !tok_list with
|(h::t) when x = h -> tok_list := t
|_ -> raise (ParseError "bad match")

let lookahead () = match !tok_list with
|[] -> None
|(h::t) -> Some h
```

S → mB | aA
A → cS | ε
B → 1#S | dB | St | Ao

```
let rec Parse_S() =
      if lookahead() = Some "m" then
            (match_tok "m"; Parse_B())
      else (* fill-in below *)
```

```
and Parse_A() =
      if lookahead() = Some "c" then (* fill-in below *)
```

```
and Parse_B() =
      if lookahead() = Some "1" then
            (match_tok "1"; match_tok "#"; parse_S())
      else (* fill-in below *)
```

5. Operational Semantics [10 pts]

A) [5 pts] Using the rules given below, show: `let x = 1 in 1 + x ⟶ 2`

In the rules, *e* refers to an expression whose abstract syntax tree (AST) is defined by the following grammar, where *x* is an arbitrary identifier and *n* is an integer.

v ::= *n*
e ::= *x* | *v* | let x = *e* in *e* | *e* + *e*

$$\text{Id}\frac{A(x) = v}{A; x \longrightarrow v} \qquad \text{Int}\frac{}{A; n \longrightarrow n}$$

$$\text{Let}\frac{A; e1 \longrightarrow v1 \quad A, x : v1; e2 \longrightarrow v2}{A; \text{let } x = e1 \text{ in } e2 \longrightarrow v2} \qquad \text{Add}\frac{A; e1 \longrightarrow v1 \quad A; e2 \longrightarrow v2 \quad v3 \text{ is } v1 + v2}{A; e1 + e2 \longrightarrow v3}$$

B) [5 pts] Below are operational semantics rules for a simple language, where the abstract syntax tree for expressions e and values *v* defined as follows.

> *v* ::= false | true
> *e* ::= v | not *e* | if *e1* then *e2*

$$\text{true}\;\frac{}{true \longrightarrow true} \qquad \text{false}\;\frac{}{false \longrightarrow false} \qquad \text{nottrue}\;\frac{e \rightarrow true}{not\ e \longrightarrow false} \qquad \text{notfalse}\;\frac{e \rightarrow false}{not\ e \longrightarrow true}$$

$$\text{Iftrue}\;\frac{\begin{array}{c} e1 \rightarrow true \\ e2 \longrightarrow v \end{array}}{if\ e1\ then\ e2 \longrightarrow v} \qquad \text{Iffalse}\;\frac{e1 \rightarrow false}{if\ e1\ then\ e2 \longrightarrow true}$$

Write a function `eval` of type `exp -> exp`, where `exp` is the OCaml representation of *e*:

```
type exp =
    Tru                         (* corresponds to true *)
  | Fals                        (* corresponds to false *)
  | If of exp * exp             (* corresponds to if e1 then e2 *)
  | Not of exp                  (* corresponds to not e *)
```

The `eval` function evaluates an expression in a manner consistent with the rules. For example:

```
eval(Tru) = Tru
eval(Not (Not Tru)) = Tru
```
etc.

```
let rec eval e =
  match e with
  | Tru -> Tru
   (* FILL IN REST *)
```

6. Lambda Calculus [13 pts]

A) [2 pts] Circle the **free variables** in the following λ-term:

`λx. y (λz.z y x) z`

B) [2 pts] Write a lambda calculus term that is *α-equivalent* to the one above.

C) [4 pts] Circle true or false for the following questions (1 point each)

True / False	The beta-normal form of `(λx.y z) z` is `y z`
True / False	The fixpoint combinator Y is used in lambda calculus to achieve recursion
True / False	A *Church numeral* is the encoding of a real number as a lambda calculus term
True / False	The expression `(λx. y) z` encodes `let x = y in z`

D) [5 pts] Reduce the following lambda expressions into beta-normal form. Show each beta reduction. If already in normal form or infinite reduction, write "normal form" or"infinite reduction", respectively.

1) `(λx. (λy. y x) (λz. x z)) (λy. y y)`

2) `(λx. x y z) (λy. z)`