CMSC330 Fall 2024 Quiz 3 Solutions

Problem 1: Context Free Grammars - Derivations [Total 6 pts]

Consider the following Grammar:

$$\begin{aligned} S &\rightarrow FSH|q \\ F &\rightarrow Ff|f \\ H &\rightarrow hHh|\epsilon \end{aligned}$$

(a) Derive `ffqhh` [4 pts]

```
S -> FSH -> FfSH -> ffSH -> ffqH-> ffqhHh -> ffqhh
```

(b) Is this an ambiguous grammar? [2 pts]

(A) Yes (B) No

$$\begin{aligned} S &\rightarrow ASB|c \\ A &\rightarrow aA|a \\ B &\rightarrow bbB|\epsilon \end{aligned}$$

(c) Derive `aacbb` [4 pts]

```
S -> ASB -> aASB -> aaSB -> aacB -> aacbbB -> aacbb
```

(d) Is this an ambiguous grammar? [2 pts]

(A) Yes (B) No

$$\begin{aligned} S &\rightarrow CSD|f \\ C &\rightarrow cC|c \\ D &\rightarrow Ddd|\epsilon \end{aligned}$$

(e) Derive `ccfdd` [4 pts]

```
S -> CSD -> cCSD -> ccSD -> ccfD -> ccfDdd -> ccfdd
```

(f) Is this an ambiguous grammar? [2 pts]

(A) Yes (B) No

$$\begin{aligned} S &\rightarrow NSP|s \\ N &\rightarrow Nn|n \\ P &\rightarrow pPp|\epsilon \end{aligned}$$

(g) Derive `nnspp` [4 pts]

```
S -> NSP -> NnSP -> nnSP -> nnsP -> nnspPp -> nnspp
```

(h) Is this an ambiguous grammar? [2 pts]

(A) Yes (B) No

Problem 2: Lexing Parsing and Evaluating

[Total 6 pts]

Given the following CFG, and assuming the **Ocaml** type system and semantics, at what stage of language processing would each expression **fail**? Mark **'Valid'** if the expression would be accepted by the grammar and evaluate successfully. Assume the only symbols allowed are those found in the grammar.

$$E \rightarrow M \text{ and } E \mid M \text{ or } E \mid M$$
$$M \rightarrow N + M \mid N - M \mid N$$
$$N \rightarrow 1 \mid 2 \mid 3 \mid 4 \mid \texttt{true} \mid \texttt{false} \mid (E)$$

	Lexer	Parser	Evaluator	Valid
`1 + 2 - (true and false)`	L	P	**E**	V
`{2}`	**L**	P	E	V
`3 * 1 - 2`	**L**	P	E	V
`2 and 5`	**L**	P	E	V
`false`	L	P	E	**V**
`true and (false)`	L	P	E	**V**
`true - (false and 2)`	L	P	**E**	V
`(5)`	**L**	P	E	V
`3 / (1 - 2)`	**L**	P	E	V
`2 + 3 - 4 or true`	L	P	**E**	V
`true or true`	L	P	E	**V**
`(true and false)`	L	P	E	**V**
`1 and 2 - (true - false)`	L	P	**E**	V
`true * 1`	**L**	P	E	V
`1 - (true or false)`	L	P	**E**	V
`true`	L	P	E	**V**
`true and (false or 3)`	L	P	**E**	V
`1 + true and false`	L	P	**E**	V
`4 / 2`	**L**	P	E	V
`(2 - true) and false`	L	P	**E**	V
`2 and 1`	L	P	**E**	V
`0 - 1`	**L**	P	E	V
`true or (false - 2)`	L	P	**E**	V

Problem 3: Operational Semantics

[Total 4 pts]

Consider the following rules for two languages. OCaml will be the meta-language for both. Take note of the order of e_1 and e_2 that is bolded in Language B.

LANGUAGE A

$$\text{(int rule)}\frac{}{A; n \Rightarrow n}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A; \text{op1}\ e_1\ e_2 \Rightarrow v_3}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A; \text{op2}\ e_1\ e_2 \Rightarrow v_3}$$

LANGUAGE B

$$\text{(int rule)}\frac{}{A; n \Rightarrow n}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A; \mathbf{e_2}\ \mathbf{e_1}\ \text{op3} \Rightarrow v_3}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A; e_1\ e_2\ \text{op4} \Rightarrow v_3}$$

Assume we derive meaning through operational semantics and read rules left-to-right. Give the Language B sentence that is semantically the same as the Language A sentence:

```
op1 3 op2 6 4
```

6 4 op4 3 op3

LANGUAGE A

$$\text{(int rule)}\frac{}{A; n \Rightarrow n}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A; \text{op1}\ e_1\ e_2 \Rightarrow v_3}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A; \text{op2}\ e_1\ e_2 \Rightarrow v_3}$$

LANGUAGE B

$$\text{(int rule)}\frac{}{A; n \Rightarrow n}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A; e_1\ e_2\ \text{op3} \Rightarrow v_3}$$

$$\frac{A; e_1 \Rightarrow v_1 \qquad A; e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A; \mathbf{e_2}\ \mathbf{e_1}\ \text{op4} \Rightarrow v_3}$$

Assume we derive meaning through operational semantics and read rules left-to-right. Give the Language B sentence that is semantically the same as the Language A sentence:

```
op2 5 op1 8 1
```

8 1 op3 5 op4

LANGUAGE A

$$\text{(int rule)}\frac{}{A;\, n \Rightarrow n}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A;\, \text{op1}\ e_1\ e_2 \Rightarrow v_3}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A;\, \text{op2}\ e_1\ e_2 \Rightarrow v_3}$$

LANGUAGE B

$$\text{(int rule)}\frac{}{A;\, n \Rightarrow n}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A;\, \mathbf{e}_2\ \mathbf{e}_1\ \text{op3} \Rightarrow v_3}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A;\, e_1\ e_2\ \text{op4} \Rightarrow v_3}$$

Assume we derive meaning through operational semantics and read rules left-to-right. Give the Language B sentence that is semantically the same as the Language A sentence:

```
op1 op2 4 3 2
```

```
2 4 3 op4 op3
```

LANGUAGE A

$$\text{(int rule)}\frac{}{A;\, n \Rightarrow n}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A;\, \text{op1}\ e_1\ e_2 \Rightarrow v_3}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A;\, \text{op2}\ e_1\ e_2 \Rightarrow v_3}$$

LANGUAGE B

$$\text{(int rule)}\frac{}{A;\, n \Rightarrow n}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 * v_2}{A;\, e_1\ e_2\ \text{op3} \Rightarrow v_3}$$

$$\frac{A;\, e_1 \Rightarrow v_1 \qquad A;\, e_2 \Rightarrow v_2 \qquad v_3 = v_1 + v_2}{A;\, \mathbf{e}_2\ \mathbf{e}_1\ \text{op4} \Rightarrow v_3}$$

Assume we derive meaning through operational semantics and read rules left-to-right. Give the Language B sentence that is semantically the same as the Language A sentence:

```
op1 op2 8 6 7
```

```
6 8 op4 7 op3
```

Problem 4: Type Checking

[Total 4 pts]

Consider the following type checking rules of OCaml:

$$\frac{}{G \vdash n : int} \qquad \frac{G(x) = t}{G \vdash x : t}$$

$$\frac{G \vdash e_1 : int \qquad G \vdash e_2 : int \qquad (+) : \texttt{int -> int -> int}}{G \vdash e_1 \ + \ e_2 : int}$$

$$\frac{G \vdash e_1 : int \qquad G \vdash e_2 : int \qquad (*) : \texttt{int -> int -> int}}{G \vdash e_1 \ * \ e_2 : int}$$

$$\frac{G \vdash e_1 : int \qquad G \vdash e_2 : int \qquad (-) : \texttt{int -> int -> int}}{G \vdash e_1 \ - \ e_2 : int}$$

$$\frac{G \vdash e_1 : t_1 \qquad G, x : t_1 \vdash e_2 : t_2}{G \vdash let\ x = e_1\ in\ e_2 : t_2}$$

Write a type checking proof for the following expression

```
let x = 5 in x + 7
```

$$\dfrac{\dfrac{}{G \vdash 5 : int} \qquad \dfrac{\dfrac{\dfrac{G, x : int(x) = int}{G, x : int \vdash x : int} \qquad \dfrac{}{G, x : int \vdash 7 : int} \quad (+) : int \to int \to int}{G, x : int \vdash x + 7 : int}}{G \vdash \text{let } x = 5 \text{ in } x + 7}$$

Write a type checking proof for the following expression

```
let x = 6 in 1 + x
```

$$\dfrac{\dfrac{}{G \vdash 6 : int} \qquad \dfrac{\dfrac{}{G, x : int \vdash 1 : int} \qquad \dfrac{\dfrac{G, x : int(x) = int}{G, x : int \vdash x : int} \quad (+) : int \to int \to int}{G, x : int \vdash 1 + x : int}}{G \vdash \text{let } x = 6 \text{ in } 1 + x}$$

Write a type checking proof for the following expression

```
let x = 2 in x * 9
```

$$\dfrac{\dfrac{}{G \vdash 2 : int} \qquad \dfrac{\dfrac{\dfrac{G, x : int(x) = int}{G, x : int \vdash x : int} \qquad \dfrac{}{G, x : int \vdash 9 : int} \quad (*) : int \to int \to int}{G, x : int \vdash x * 9 : int}}{G \vdash \text{let } x = 2 \text{ in } x * 9}$$

Write a type checking proof for the following expression

```
let x = 42 in 0 - x
```

$$\dfrac{\dfrac{}{G \vdash 42 : int} \qquad \dfrac{\dfrac{}{G, x : int \vdash 0 : int} \qquad \dfrac{\dfrac{G, x : int(x) = int}{G, x : int \vdash x : int} \quad (-) : int \to int \to int}{G, x : int \vdash 0 - x : int}}{G \vdash \text{let } x = 42 \text{ in } 0 - x}$$