I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination

Signature: ____________________________

Ground Rules

- You may use anything on the accompanying reference sheet anywhere on this exam
- Please write legibly. **If we cannot read your answer you will not receive credit**
- You may not leave the room or hand in your exam within the last 10 minutes of the exam
- If anything is unclear, ask a proctor. If you are still confused, write down your assumptions in the margin

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
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<td>Total</td>
<td>100 + 5</td>
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Problem 1: Language Concepts

Indicate True or False for each of the below statements. 1 point per question.

(A) OCaml and Rust's static type system force each function or data structure to be created for only one specific type of data making it hard to create generalized code in those languages.  
True False

(B) While Rust is a "memory-safe" language, OCaml and Python are not "memory-safe" and have features which may lead to memory corruption.  
True False

(C) Any language described by a CFG can also be described by a regular expression.  
True False

(D) Lambda Calculus can mimic the behavior of a Finite State Machine.  
True False

(E) Python associates types with values, not variables.  
True False

(F) A Context Free Grammar is Ambiguous if there is a single left most and single right most derivation for all strings of literals in the grammar.  
True False

(G) "Double Freeing" (repeatedly de-allocating the same memory) is prevented in Rust by its semantics and compiler.  
True False

(H) Operational Semantics specifies the meaning of language syntax such as whether a = 7 means "check for equality" or "bind this value to this variable".  
True False

(I) While Python and OCaml often share data between different parts of the program, Rust must copy data often as sharing pointers to the same block of memory is not permitted for safety reasons.  
True False

(J) Due to its limited nature, the simply typed Lambda Calculus we studied is not powerful enough to represent ALL types of data or programs available in "real" programming languages.  
True False

Indicate which of the following choices best answers the question given. 2 points per answer.

(K) Python programs with type problems such as dividing an integer by a string are usually detected...  
1 pts

A The semantics the language prevent this kind of problem from happening
B At Compile Time before a program is created
C At Run Time when code causing the problem executes
D This type of error is not detected and can lead to memory corruption.

(L) OCaml programs with memory problems such a using memory after it has been de-allocated are detected...  
2 pts

A The semantics the language prevent this kind of problem from happening
B At Compile Time before a program is created
C At Run Time when code causing the problem executes
D This type of error is not detected and can lead to memory corruption.

(M) Rust programs with memory problems such a using memory after it has been de-allocated are detected...  
2 pts

A The semantics of the language prevent this kind of problem from happening
B At Compile Time before a program is created
C At Run Time when code causing the problem executes
D This type of error is not detected and can lead to memory corruption.
Problem 2: Regular Expressions

(a) Which of the following strings are accepted by the regular expression below?

\[ L^*DS | [OB] \{2 \} \]

Select NONE if none of the first five (5) options match.

- DS
- OBOB
- DOL
- LSBB
- OB
- NONE

(b) Write a Regular Expression

Consider the OCaml variant:

```ocaml
type shape = Rect of int * int | Circ of float
```

Write a regular expression that would describe an OCaml expression which binds a shape to a variable.

EXAMPLES:

<table>
<thead>
<tr>
<th>Valid</th>
<th>Invalid</th>
</tr>
</thead>
<tbody>
<tr>
<td>let x = Rect(10,2)</td>
<td>let x = Rect 10,2 (* need parens around constructor *)</td>
</tr>
<tr>
<td>let y = Circ(1.23)</td>
<td>let yz = Circ(1.2) (* variable name too long *)</td>
</tr>
<tr>
<td>let z = Rec(0,-10)</td>
<td>let a = Rect(10.2,5.6) (* Rect requires ints in its pair, not floats *)</td>
</tr>
<tr>
<td>let w = Circ(-1.2)</td>
<td>let B = Circ(-1.2) (* variable name uppercase *)</td>
</tr>
</tbody>
</table>

Constraints and limitations:

1. The variable name will be a single lowercase alphabetic character
2. One or more spaces can appear between the `let` keyword and the variable name
3. Zero or more spaces can appear on either side of the equal sign `=` in the strings
4. The constructor of the shape must use parenthesis.
5. No spaces may appear within the parentheses `()` of the constructor
6. Multi-digit numbers MAY have leading zeros (i.e. 0123 is accepted)
7. Only unary negation of numbers is accepted, NOT unary plus.
8. Floating point values always have a decimal point with trailing digits.

```ocaml
let +[a-z]* = *(Rect\([-\]d+\][-\]d+)| Circ\([-\]d+\].d+|\d+)
```

Problem 3: Context Free Grammars

Consider the following Grammar:

- S -> S is S | U
- U -> Not U | P
- P -> That | It

Derive the string "That is Not It". Use a leftmost derivation and show all steps for full credit

```
S \rightarrow S is S \rightarrow U is S \rightarrow P is S \rightarrow That is S \rightarrow That is U \rightarrow That is Not U \rightarrow That is Not P \rightarrow That is Not It
```
Problem 4: Finite State Machines

Using the Subset Construction algorithm, convert the following NFA to a DFA, and fill in the blanks appropriately matching the DFA provided with the right nodes and transitions. Only answer in the blank boxes / multiple choice below will be graded.

NFA:

DFA:

(a) Blanks

(b) Final States
Problem 5: Language Analysis

Consider the following two code fragments which attempt to compute similar results involving only integers or integer tuples.

1. # Python
   2. def ratio(x,y,as_tup):
   3.     if as_tup:
   4.         return (x,y,x//y)
   5.     else:
   6.         return x//y

1. (* OCaml *)
   2. let ratio x y as_tup =
   3.     if as_tup then
   4.         (x,y,x/y)
   5.     else
   6.         x/y

(a) **Python Correctness:** What would the Python version do on an input like `ratio(15,3,True)` vs `ratio(15,3,False)`? [3 pts] Would it function correctly?

It would return a tuple of (15,3,5) for True and just 5 for False. It would function correctly in both cases.

(b) **OCaml Correctness:** The OCaml version will not compile. Why not? Describe the problems the compiler will identify in a few sentences.

The true branch returns an int*int*int tuple while the false branch returns just an int. This will not type check so the compiler rejects it.

(c) **OCaml Equivalence:** Describe how to get the same effect in the OCaml function as is present in the Python program so that the same basic data can be returned in the OCaml version. Mention any relevant features of OCaml that would be useful here.

Establish a new Variant data type that can carry either an int*int*int or just an int, something like type ratio_t = Int of int — Tup of int*int*int;;
Then modify the ratio function to use one or the other in the true/false branch that the return type is ratio_t.

(d) **Rust Safety:** Nearby is a partial Rust version of the `ratio()` function along with `main()` that calls it several times. Describe whether the Rust compiler will compile this program and the results of running it.

The code is syntactically correct and the compiler will produce a runnable program. While running, a Panic / Error will occur due to division by zero stemming from the call at line 7 with 0 as a denominator.
Consider the following operational semantics for some list operations with OCaml as the metalanguage:

\[
\begin{align*}
\text{[]} & \Rightarrow \text{[]} \\
\text{[x, y]} & \Rightarrow x :: [y] \\
\text{len([])} & \Rightarrow 0 \\
\text{len([x])} & \Rightarrow x :: [] \\
\text{len([x, y, z])} & \Rightarrow x :: [y, z] \\
\text{v} & \Rightarrow h :: t \\
\text{len(t)} & \Rightarrow l \\
\text{n} & \Rightarrow l + 1 \\
\text{len(v)} & \Rightarrow n \\
\text{v.get(0)} & \Rightarrow x \\
\text{v.get(x)} & \Rightarrow y :: ys \\
\text{ys.get(z)} & \Rightarrow n \\
\text{len(v)} & \Rightarrow l \\
\text{v.get(x)} & \Rightarrow None
\end{align*}
\]

Construct a proof that \( \text{len([1, 2])} \Rightarrow 2 \). Use the space below and follow the structure of the tree provided. Letters indicate blanks that must be filled in.
Problem 7: Rust Ownership

Nearby is a short Rust snippet. When compiled it will give an error to the effect that Line 3: `borrowed value &n does not live long enough`. Explain in a two sentences why this error is occurring:

- After the function ends, the owner 'n' goes out of scope. Thus, the reference to 'n' that is being returned is invalid.

Explain code changes that would fix the `append_one()` function and allow it to append 1 to vectors. Your changes may include alterations to the function and `main`.

```rust
fn append_one(v: &mut Vec<&i32>) {
    let n = 1;
    v.push(&n);
}
```

Problem 8: Type Inference

Give the type inferred for the following OCaml expressions.

```ocaml
fun a b c -> (* A *)
    (List.map a c) > b || c > b

('a -> 'a) -> 'a list -> 'a list -> bool

fun x y z -> (* B *)
    let _ = List.fold_left x
    (0,0)
    [true;false;y]
    in z

(int * int -> bool -> int * int) -> bool -> 'c -> 'c

fun a b c d -> (* C *)
    List.fold_left a b (map c d)

('b->'c->'b) -> 'b -> ('d->'c) -> 'd list -> 'b
```
Problem 9: OCaml Coding

Below are types and a description of the less_traveled_by function. Analyze the examples provided and implement this function in OCaml.

```
type path = (* Branching paths with travel counts along Roads *)
  | Road of int*path (* count of travel along Road AND remaining path *)
  | Diverge of path*path (* left path AND right path *)
  | End;; (* end of the path *)

(* EXAMPLE 1 *)
# let road_to_nowhere = End;;
# less_traveled_by road_to_nowhere;;
- : string list * int = ([], 0)

(* EXAMPLE 2 *)
# let cormack_road = Road(1, Road(2, Road(5, Road(8, End))));;
# less_traveled_by cormack_road;;
- : string list * int = ([], 16)

(* EXAMPLE 3 *)
# let branching = Diverge( Diverge( Road(4,End), (* [["left","left","left"],4] *)
                                   Road(2,End)), (* [["left","left","right"],2] <= LEAST TRAVELED *)
                             Road(3,End)), (* [["left","right"],3] *)
                     Diverge( Road(7,End), (* [["right","left"],7] *)
                             Road(6,End)));
# less_traveled_by branching;;
- : string list * int = (["left; "left; "right"], 2)

(* EXAMPLE 4 *)
# let yellow_wood = Road(2, Diverge( Road(4, Road(1, Diverge( Road(3, End), (* [["left","left"], 10] *)
                                       End)), (* [["left","right"], 7] *)
                               Road(1, Diverge( Diverge( Road(5, End), (* [["right","left","left"], 8] *)
                                                 Road(2, End)), (* [["right","left","right"], 5] LEAST*)
                                           Road(1, Road(4, End))))));
# less_traveled_by yellow_wood;;
- : string list * int = (["right; "left","right"], 5)
```

Write your implementation on the next page.
Write your implementation here.

(* Returns a pair of (string list, int); the list shows one sequence of left/right choices through the path with smallest total count along roads in the path. *)

let rec less_traveled_by path =

SOLUTION:

let rec less_traveled_by path =
    match path with
      | End -> ([],0)
      | Road(count,rem) ->
        let (path, total) = less_traveled_by(rem) in
          (path, count+total)
      | Diverge(left,right) ->
        let (lpath, ltotal) = less_traveled_by(left) in
        let (rpath, rtotal) = less_traveled_by(right) in
          if ltotal < rtotal then
            ("left"::lpath, ltotal)
          else
            ("right"::rpath, rtotal)
    ;;
Problem 10: Lambda Calculus

(a) **Lazy Evaluation, Single Step** Perform a single step of Beta Reduction using the Lazy / Call by Name Evaluation Strategy on the given Lambda Calculus expression. If the expression cannot be reduced, select “Beta Normal Form”. The options offered may be alpha equivalent to what you calculated.

\[(\lambda x. xx)((\lambda y. yy)(\lambda z. zz))\]  
\[(\lambda x. (\lambda y. y z) a)\]

- **A** \((\lambda x. xx)(\lambda z. zz)\)
- **B** \((\lambda x. xx)((\lambda z. zz)(\lambda z. zz))\)
- **C** \(((\lambda y. yy)(\lambda z. zz))((\lambda y. yy)(\lambda z. zz))\)
- **D** Beta Normal Form
- **E** None of the above

(b) **Eager Evaluation, Single Step**: As before, perform a single step of Beta Reduction but this time use the Eager / Call by Value Evaluation Strategy.

\[(\lambda x. xx)((\lambda y. yy)(\lambda z. zz))\]  
\[(\lambda x. (\lambda y. y z) a)\]

- **A** \((\lambda x. xx)(\lambda z. zz)\)
- **B** \((\lambda x. xx)((\lambda z. zz)(\lambda z. zz))\)
- **C** \(((\lambda y. yy)(\lambda z. zz))((\lambda y. yy)(\lambda z. zz))\)
- **D** Beta Normal Form
- **E** None of the above

(c) **Reduce to Normal Form**: Convert the following to Beta Normal Form: \((\lambda x. \lambda y. \lambda z. x y z)\ a\ b\ (\lambda d. d\ c)\ e\)

- **A** \(a b e c\)
- **B** \(a b (\lambda d. d\ c)\)
- **C** \(a b\)
- **D** \(a b e c\ y\ z\)
- **E** Already in Beta Normal Form
- **F** Diverges like infinite recursion
- **G** None

(d) **Reduce to Normal Form**: Convert the following to Beta Normal Form: \((\lambda y. y\ y\ y)(\lambda y. y\ y\ y)\)

- **A** \((\lambda y. y\ y\ y)\)
- **B** \(y\ y\ y\)
- **C** \(y\)
- **D** \((\lambda y. y\ y\ y)(\lambda y. y\ y\ y)\)
- **E** Already in Beta Normal Form
- **F** Diverges like infinite recursion
- **G** None

The blank area below may be used for calculations.

Problem 11: Extra Credit

(a) What is your TA’s name and what is your section id?  

Look it up on the website!

(b) What is your favorite pun?  

What did the duck say when the waiter gave him the check? “Put it on my bill.”