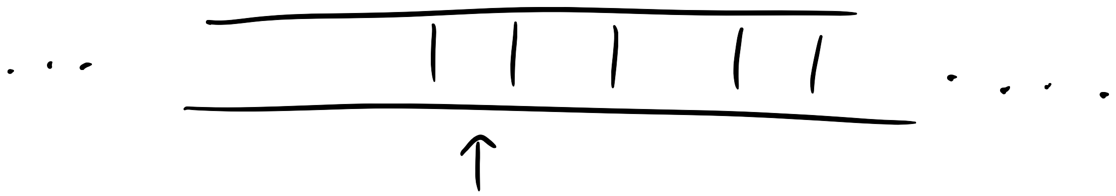
CMSC 330 10/24/24

Lambda Calc



$E \to x$ → same as y, z, etc.

$\quad | \ \lambda x . E$ → function w/ expression

$\quad | \ E \ E$ → application

$\quad | \ (E)$ → precedence

lambda marks function

parameter

$\lambda y . x$

output

fun $y \to x$

$(\lambda y . \lambda x . x)$

Scope of parameters extends all the way

fun $y \to$ fun $x \to x$

β-reduction : reducing functions using application ex

$$(\lambda y . y) \; x$$

$$x$$

$a \; b$

$a \; (\lambda x . x)$ in beta-normal form

$$(a \; b) \; c$$

$$(\lambda x . x) (\lambda y . y) (\lambda z . z)$$

$$(\lambda y . y) (\lambda z . z)$$

$$(\lambda z . z)$$

$$(\lambda x . \; x \; (\lambda y . \; y \; (\lambda z . z)))$$

$$(\lambda x . (a \; b) \; c)$$

$$(\lambda x . x x) (\underbrace{\lambda y . y y}_{x})$$

$$x \; x \iff (\lambda y . y y) (\lambda y . y y)$$

$(\lambda x.\ \underline{(\lambda y.\ y\,y)\ (\lambda y.\ y\,y)})\ a$

lazy eval : process outside first

eager eval : simplify expr in function before evaluating / application

$(\lambda x.\ a\ x\ a)\ (\underbrace{(\lambda y.\ y\,y)\ z})$

$a\ ((\lambda y.\ y\,y)\ z)\ a$ lazy

$\underline{a\ (z\ z)\ a}$

$(\lambda x.\ a\ x\ a)\ (z\ z)$ eager

$\underline{a\ (z\ z)\ a}$

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free variables → not attached to a value

bound variables → attached to value

$(\lambda x.\ x\ a)\ b$

$(\lambda x.\ \underline{\lambda y.\ x\ a\ y})\ b$

$(\lambda x. \lambda x. x)\ a$

$(\lambda x. x)$

α-conversion

fun x → (fun x → x)

α-equivalence → semantically equivalent expressions

$(\lambda y. (\lambda x . x))\ a$

$(\lambda y. (\lambda x. x))\ b$ ✗

useless

$(\lambda y. (\lambda x. y))\ x \rightarrow (\lambda y. (\lambda a. y))\ x$

$(\lambda a. x)$

=

$(\lambda y. x)$ ✓ $(\lambda x. x)$ ✗

Church Encodings

True : $\lambda x. \lambda y. x$

False : $\lambda x. \lambda y. y$

if a then b else c : a b c

ex if true then false else true

$$(\lambda x.\lambda y.x)(\lambda x.\lambda y.y)(\lambda x.\lambda y.x)$$

$$(\lambda y.(\lambda x.\lambda y.y))(\lambda x.\lambda y.x)$$

$\Updownarrow$ α-equivalent

$$(\lambda a.(\lambda x.\lambda y.y))(\lambda x.\lambda y.x)$$

$$(\lambda x.\lambda y.y) = \text{false}$$

$$(\lambda x.\lambda y.x)(\text{false})(\text{true})$$

$$(\lambda y.\text{false})(\text{true})$$

$$\text{false}$$